The geostrophic turbulence regime of rotating convection.

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Outline

I. Thermal convection and the use of radiative heating

- Rayleigh-Bénard convection
- Radiatively heated convection

II. Impact of global rotation on a convective flow

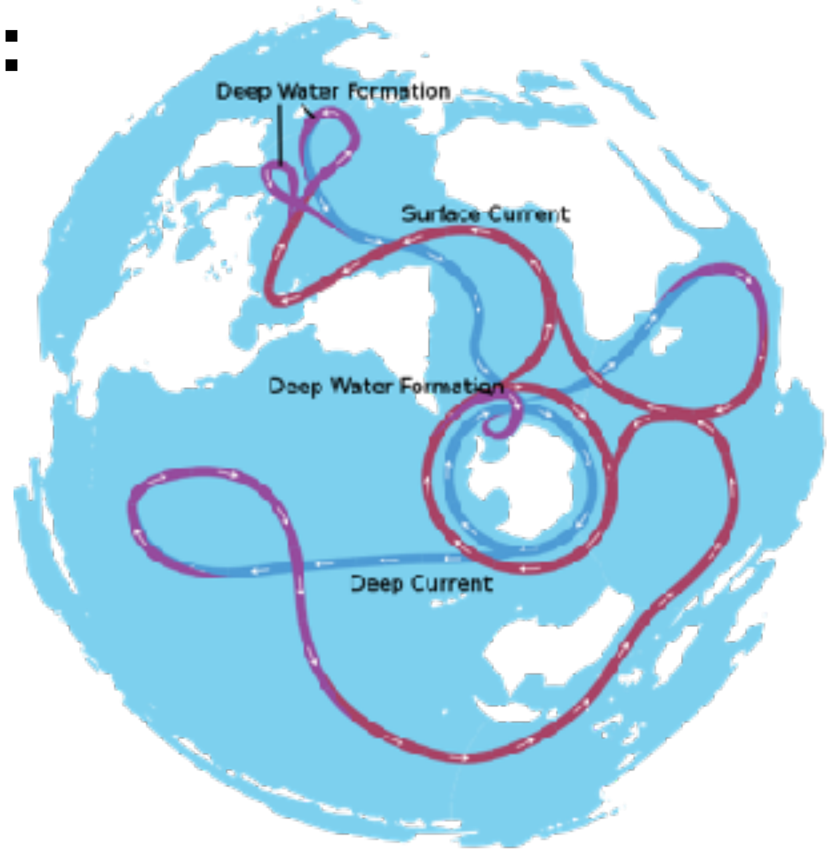
- Rotation suppresses vertical transport: scaling predictions
- Laboratory data on rotating RB convection

III. Radiative heating to observe the Geostrophic Turbulence regime

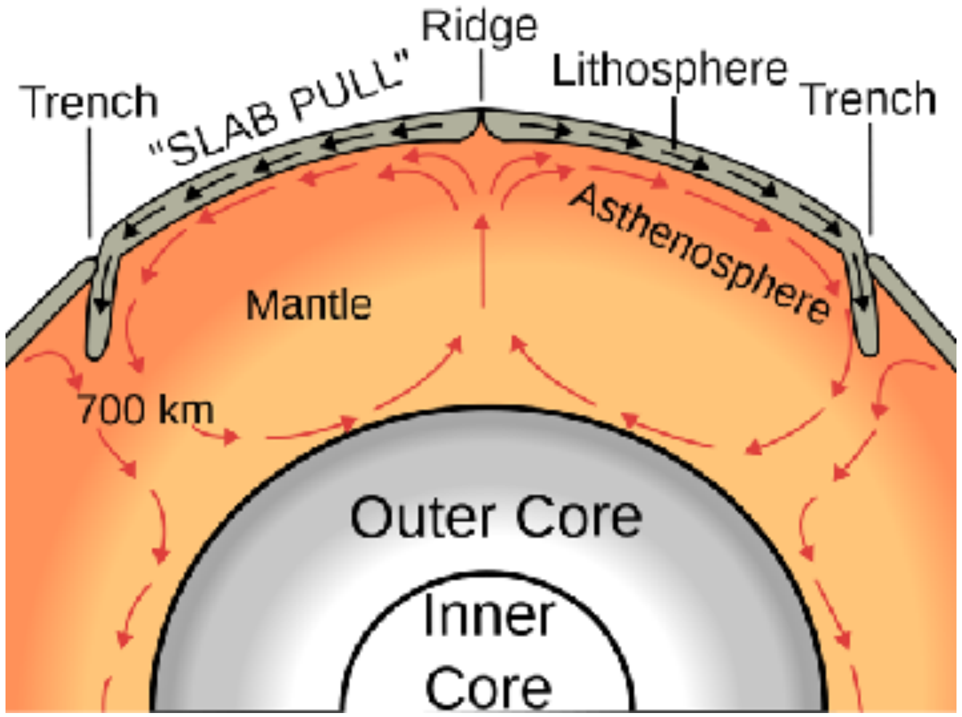
- Experimental setup
- Fully turbulent behavior
- First observation of the GT regime

Natural convective flows

Oceans:

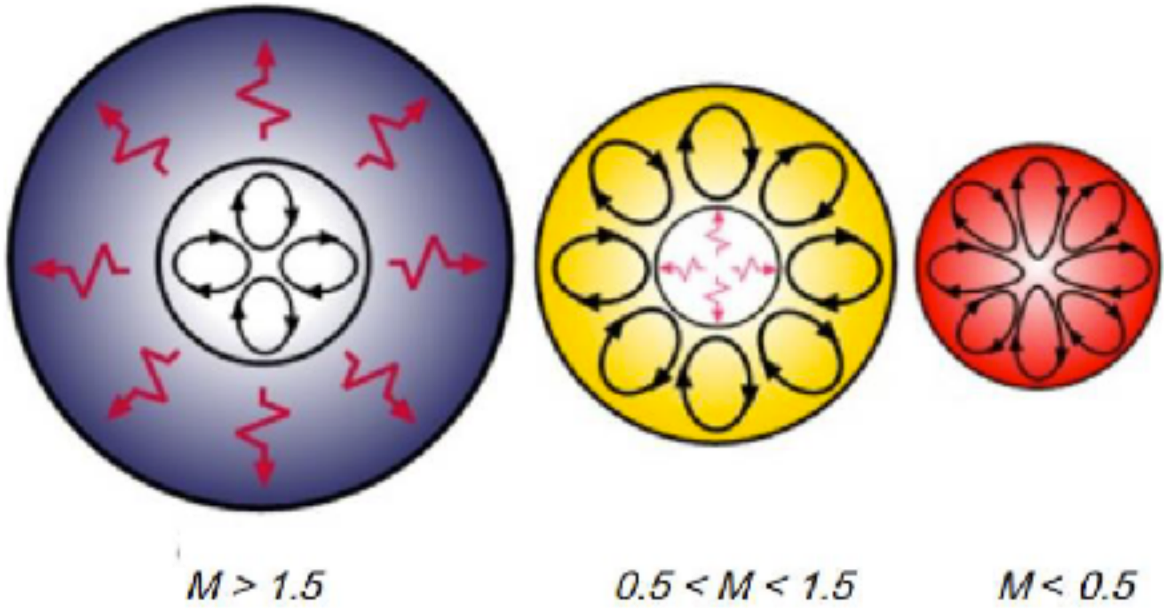


Planetary interiors:

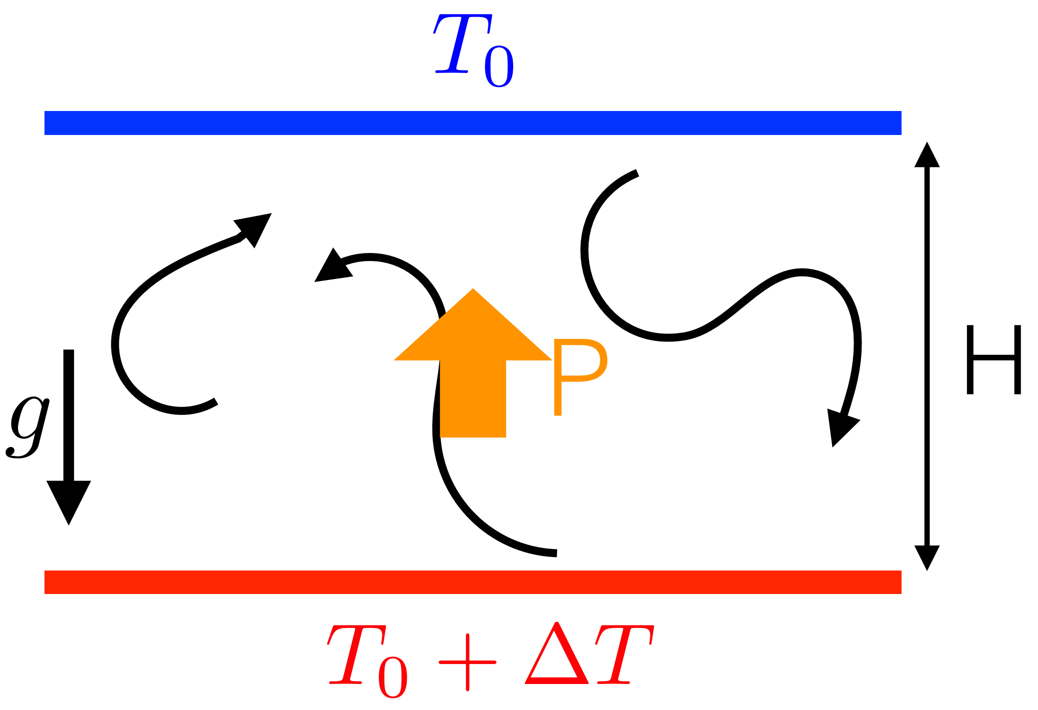


Clouds:

Stars:



Rayleigh-Bénard convection



Enhanced heat transport:

$$P = f(\Delta T)\ \ ?$$

Dimensionless parameters:

$$Nu = \frac{PH}{\lambda \Delta T}$$

$$Ra = \frac{\alpha g \Delta T H^3}{\kappa \nu}$$

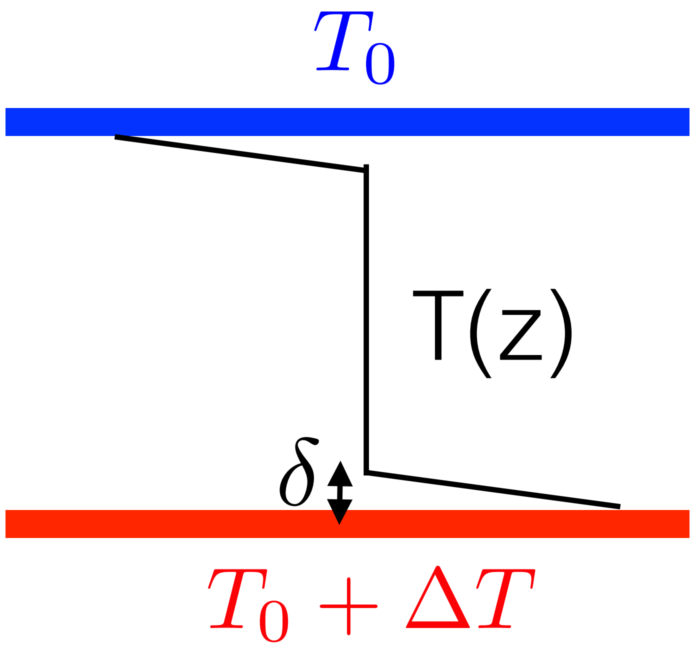
$$Pr = \frac{\nu}{\kappa}$$

$$\boxed{Nu \sim Ra^{\gamma}\, Pr^{\chi}}$$

Scaling-law that can be extrapolated to parameter values of natural flows.

Two competing predictions

- $\gamma = 1/3$ [Malkus (1954)]



Marginally stable BLs:
δ independent of H.
P independent of H.

$$Nu \sim Ra^{1/3}$$

most existing
experimental RB data

- $\gamma = 1/2$ [Spiegel, Kraichnan (1962)]

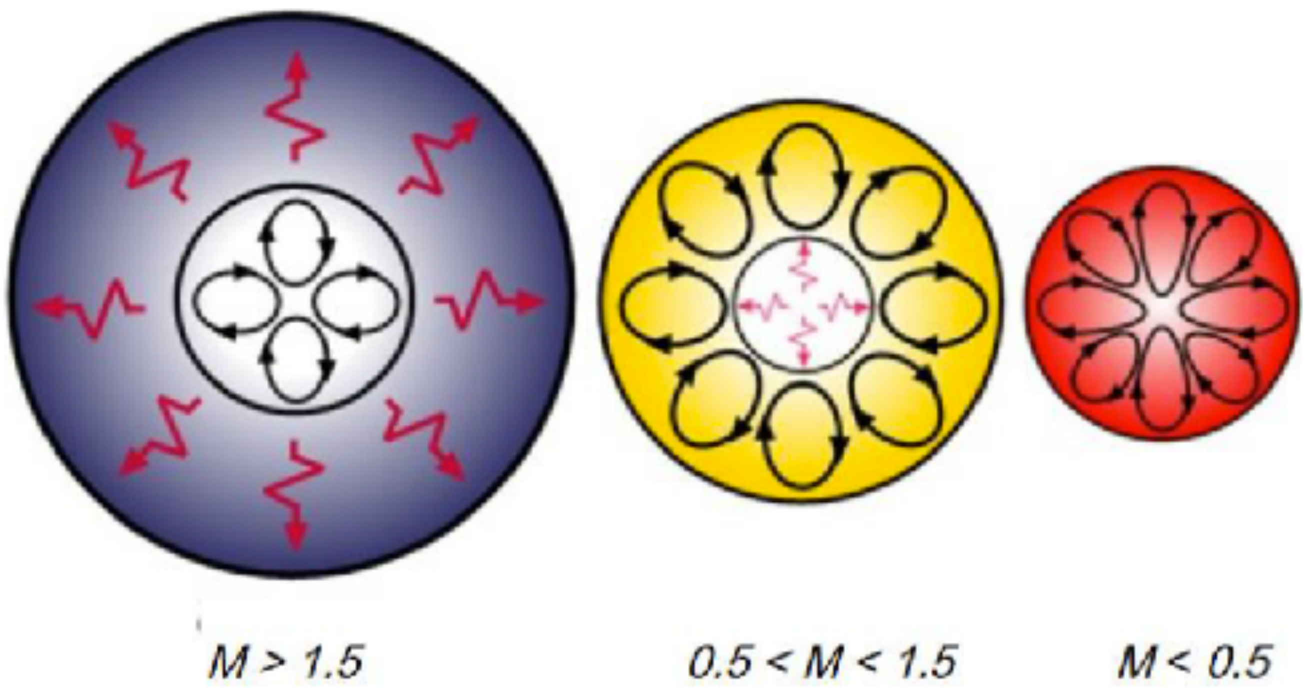
~~Heat flux should be independent of molecular diffusivities κ and ν.~~

$$\cancel{Nu \sim Ra^{1/2}\, Pr^{1/2}}$$

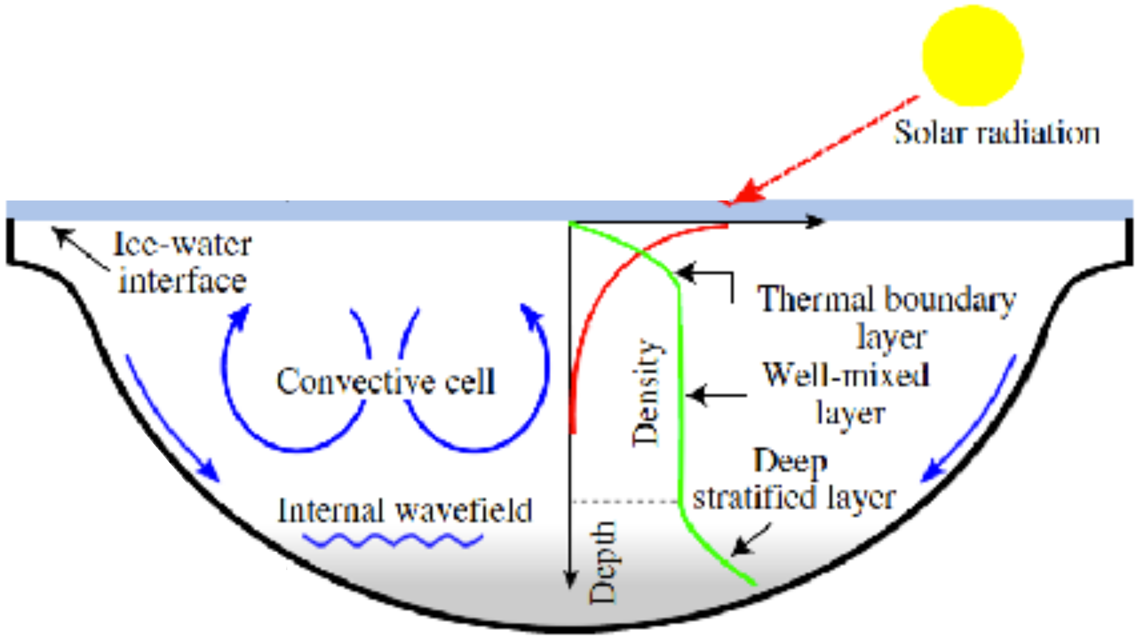
mixing-length or
« ultimate » regime

Radiatively driven convection

Rayleigh-Bénard boundary layers are irrelevant to many natural flows:



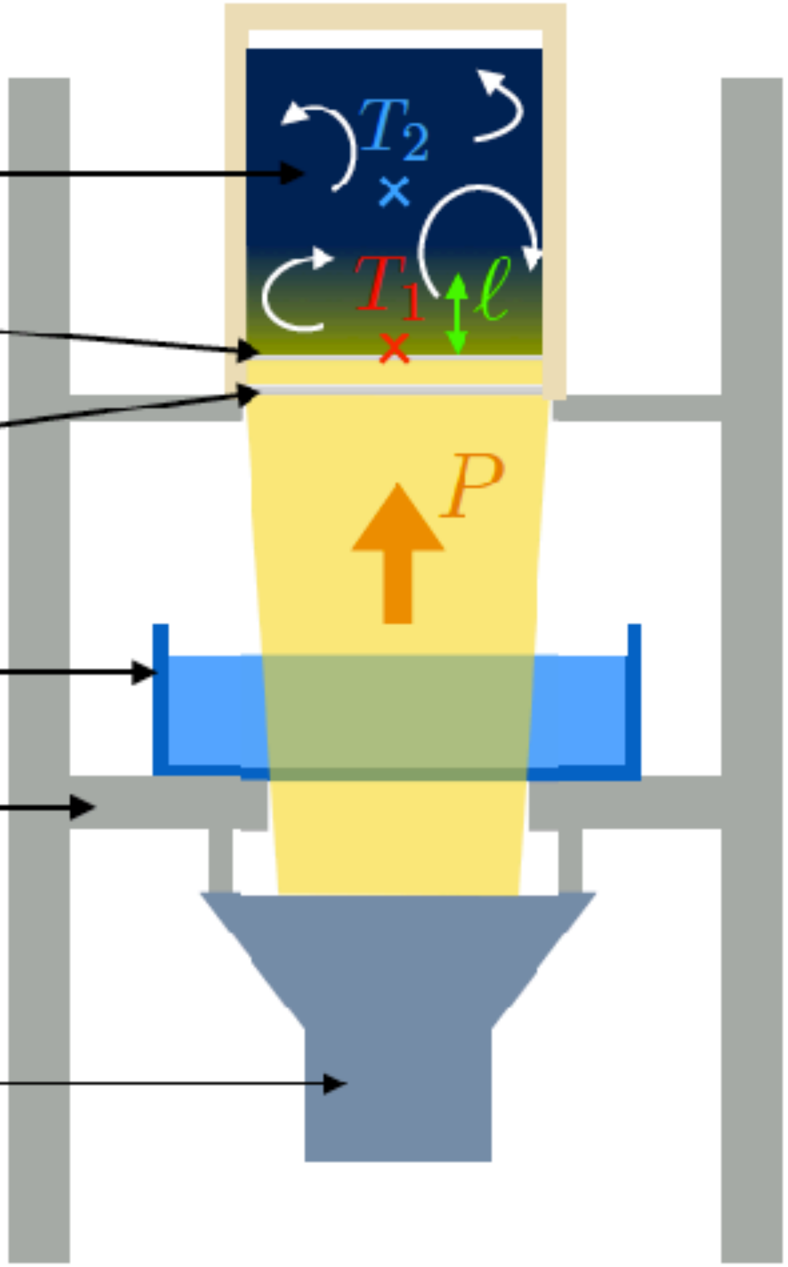
Stellar convection



Frozen lakes
in the spring

Heat input directly into the bulk turbulent flow:
Can radiative heating bypass the boundary layers and produce the ultimate scaling regime in a lab experiment?

Radiatively driven convection in the lab



water + dye

sapphire plate

borosilicate plate

water-cooled
IR filtration stage

water-cooled
thermal screen

high-throughput
spotlight

What about cooling?

- A cooling plate would induce boundary layers and a regime similar to Rayleigh-Bénard.
- **Secular heating:** we do not cool down the fluid.

$$\partial_t T + \mathbf{u} \cdot \boldsymbol{\nabla} T = \kappa \boldsymbol{\nabla}^2 T + \frac{P}{\rho C \ell} e^{-z/\ell}$$

space average

$$\rho C H \frac{\mathrm{d}\overline{T}(t)}{\mathrm{d}t} = P$$

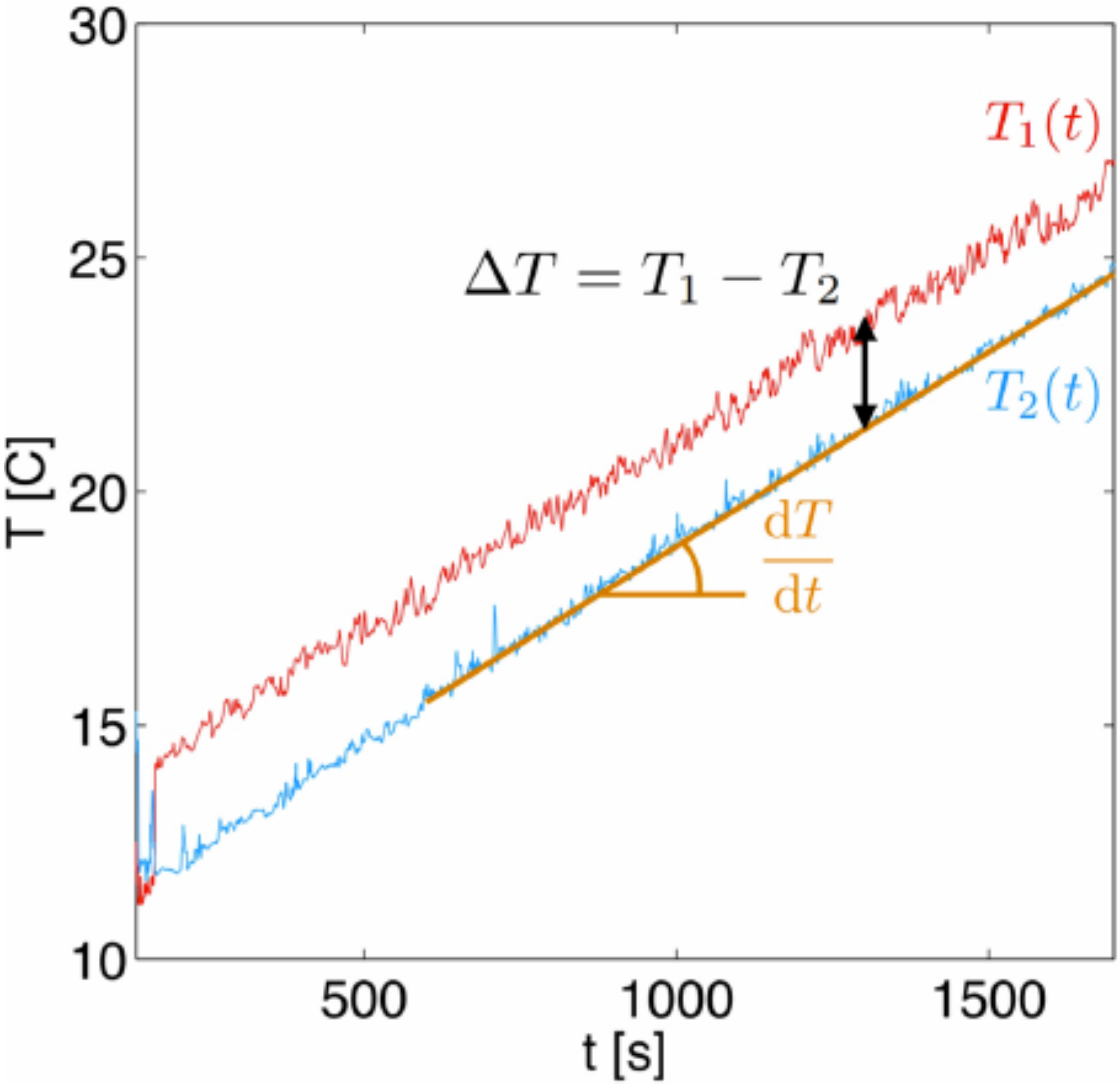
temporal drift of the mean T
(slopes gives value of P)

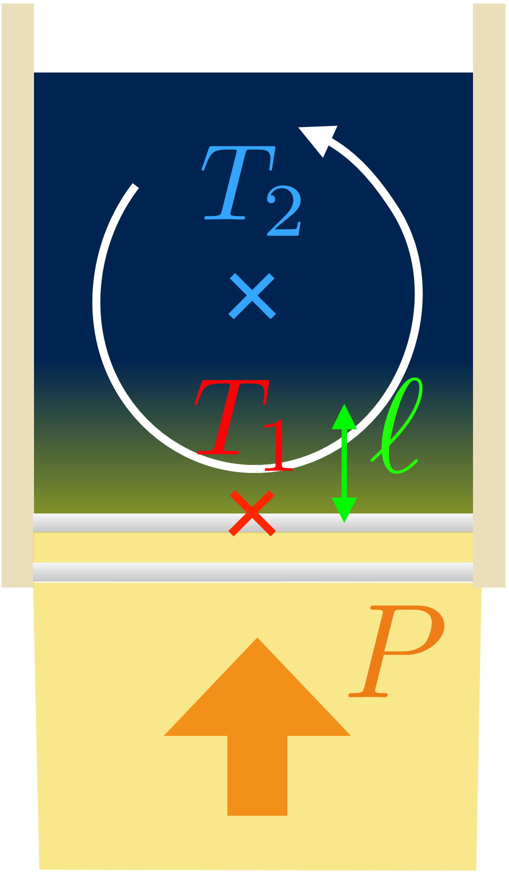
internal gradients:

$$\theta(\mathbf{x}, t) = T(\mathbf{x}, t) - \overline{T}(t)$$

$$\partial_t \theta + \mathbf{u} \cdot \boldsymbol{\nabla} \theta = \kappa \boldsymbol{\nabla}^2 \theta + \frac{P}{\rho C} \left(\underbrace{\frac{e^{-z/\ell}}{\ell}}_{\text{radiative heating}} - \underbrace{\frac{1}{H}}_{\text{uniform cooling}} \right)$$

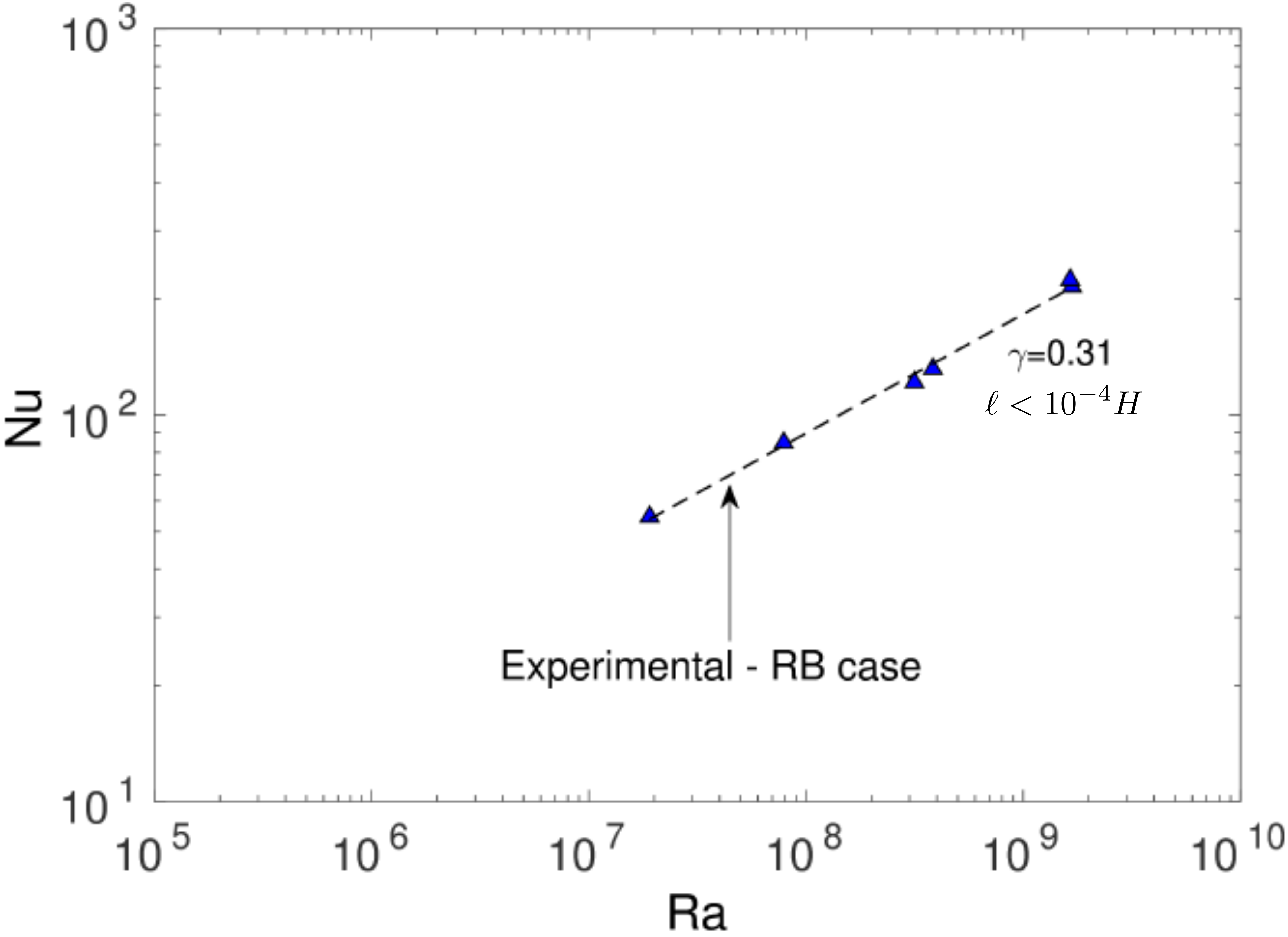
Experimental demonstration



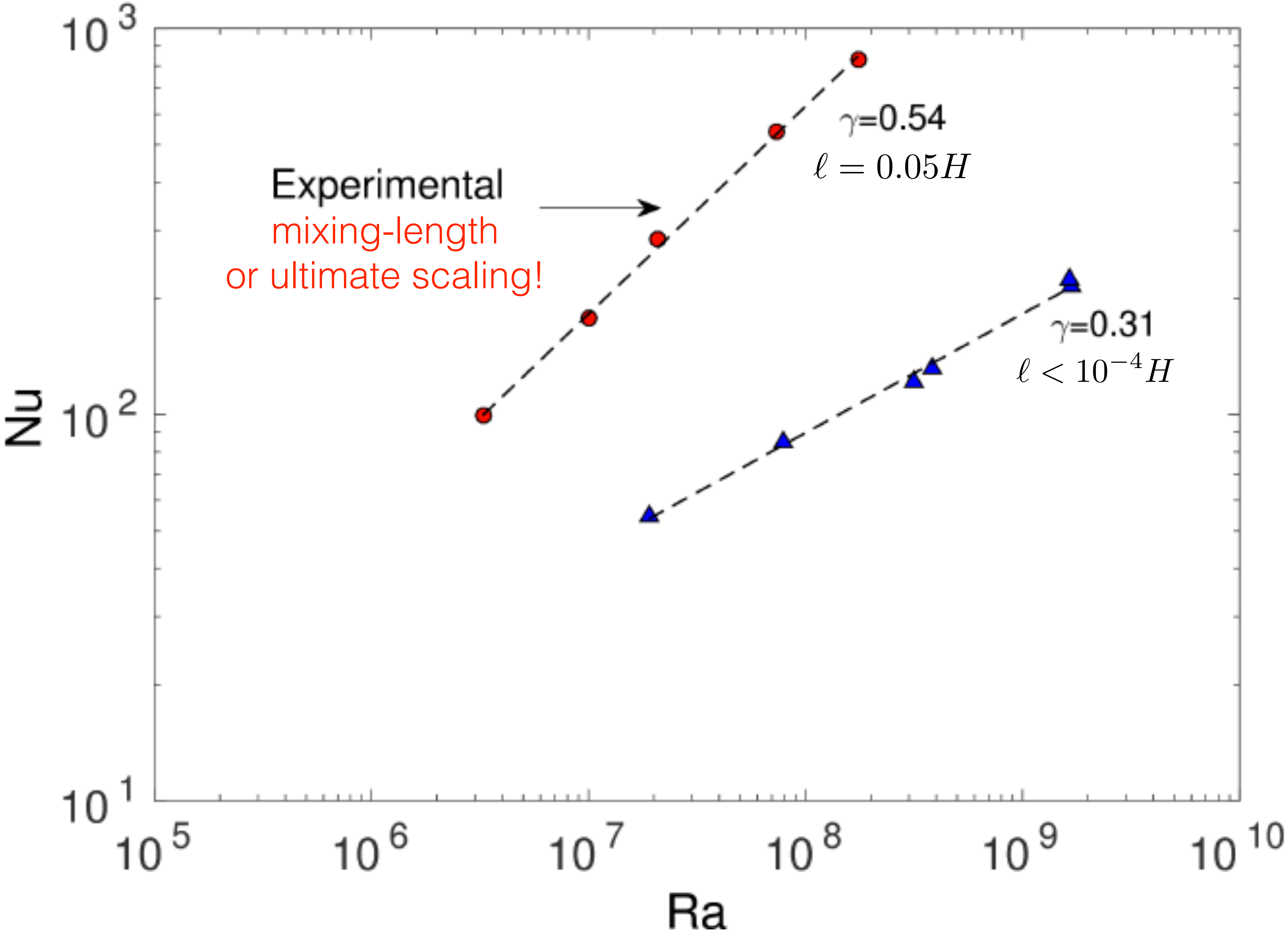


$\Delta T = T_1 - T_2 = \theta_1 - \theta_2$ is stationary around room T.

Nusselt vs Rayleigh



Nusselt vs Rayleigh

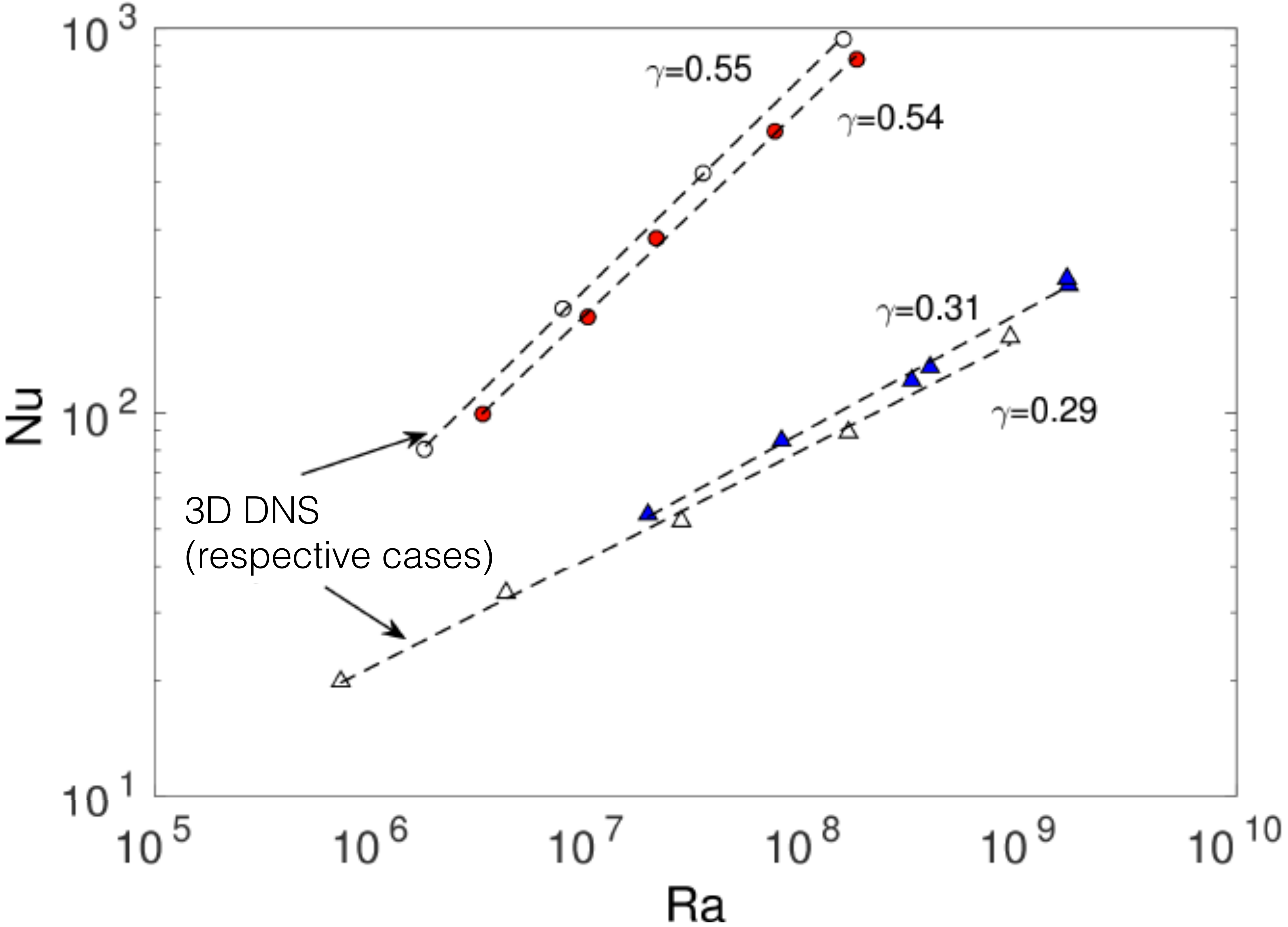


Experimental

mixing-length

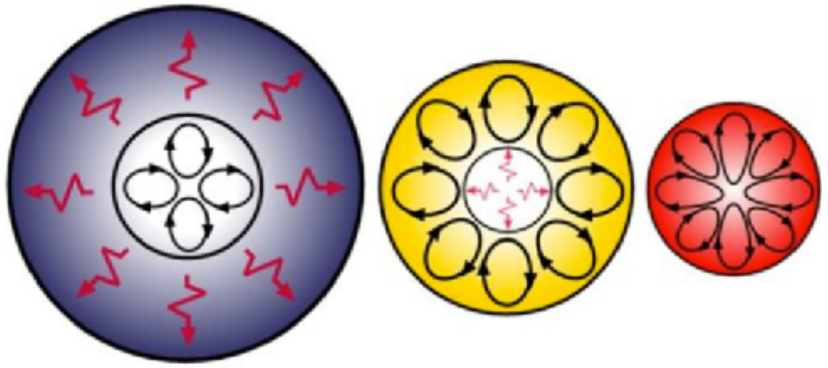
or ultimate scaling!

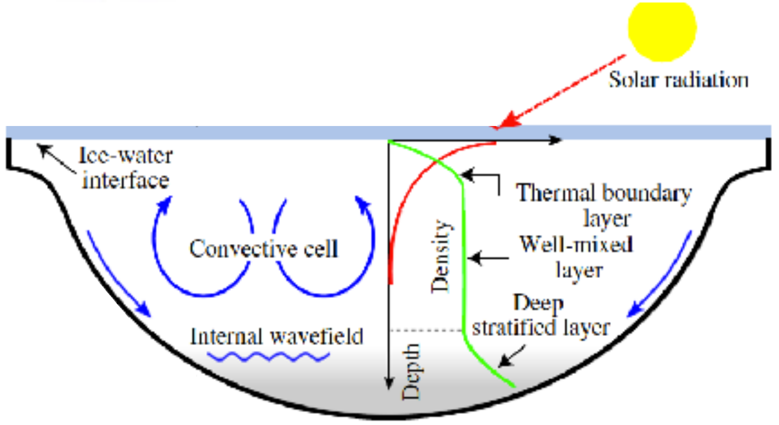
Nusselt vs Rayleigh



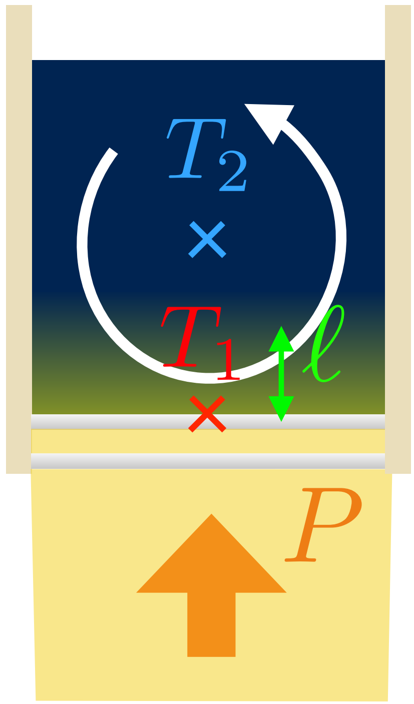
3D DNS
(respective cases)

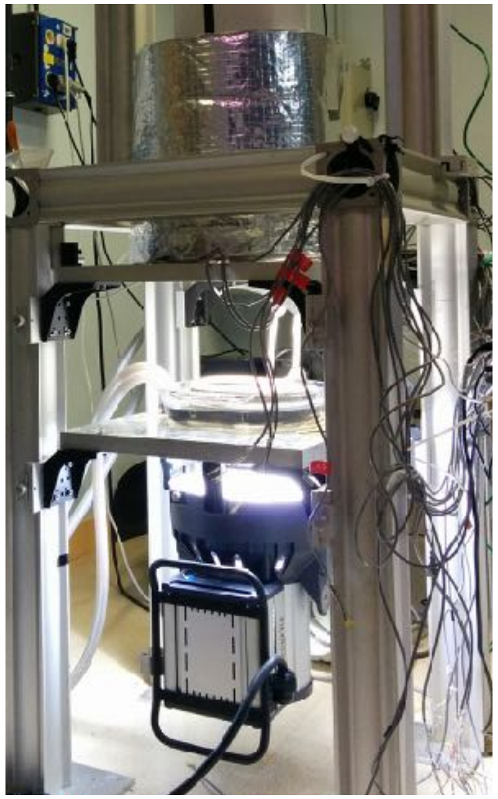
Summary I

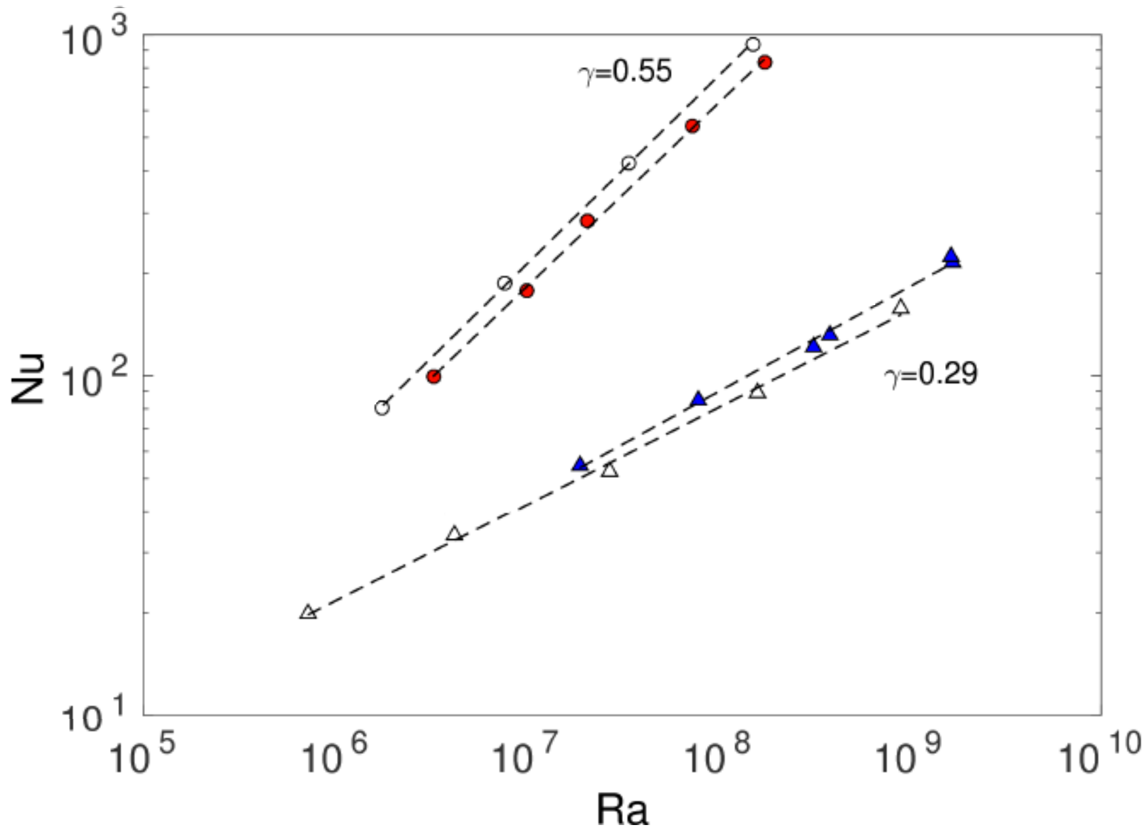




- A clear observation of the ultimate scaling-regime.
- Bridges the gap between astro models and lab experiments.
- Improved models of nonlocal and/or penetrative convection?







[S. Lepot, S. Aumaître, B. Gallet, PNAS, 2018]

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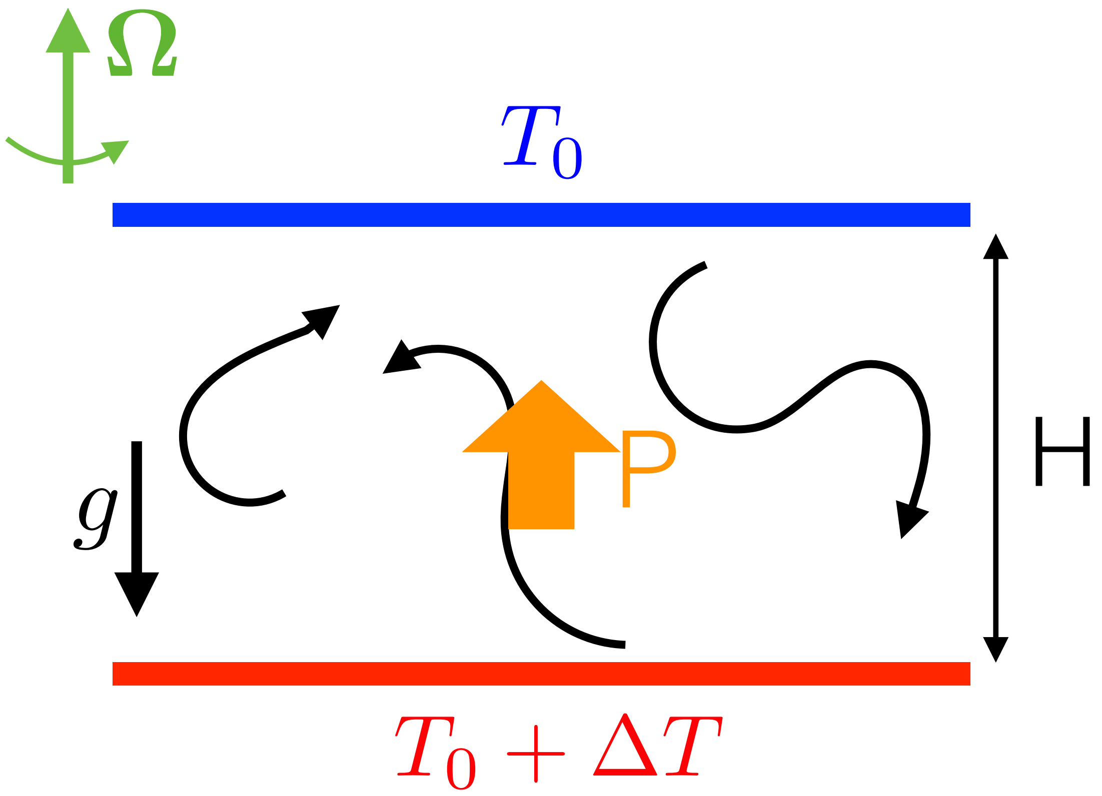
Rotating RB convection



Enhanced heat transport:

$$P = f(\Delta T, \Omega)?$$

Dimensionless parameters:

$$Nu = \frac{PH}{\lambda \Delta T}$$

$$Ra = \frac{\alpha g \Delta T H^3}{\kappa \nu}$$

$$Pr = \frac{\nu}{\kappa}$$

$$E = \frac{\nu}{\Omega H^2}$$

$$\boxed{Nu \sim Ra^{\gamma}\, Pr^{\chi}\, E^{\xi}}$$

One additional parameter: makes dimensional analysis more challenging…

Rotation suppresses convection

- Rotation delays threshold for convection:

$$Ra_c \sim E^{-4/3} \qquad \text{[Chandrasekhar]}$$

+ scaling-laws for the E-dependence of horizontal scale of the flow, vertical scale, temperature over velocity ratio, etc.

- This ordering remains valid at finite distance from threshold provided $E \ll 1$.

- Motivates the derivation of reduced asymptotic equations valid for $E \ll 1$, through expansion in $E^{1/3}$.

[Julien et al. 1998]

- Crucial point: the only control parameter entering the reduced system is

$$\tilde{R}a = Ra\, E^{4/3} \sim \frac{Ra}{Ra_c} \quad \Rightarrow \quad \boxed{Nu = \mathcal{F}(Ra\, E^{4/3}, Pr)}$$

Bulk versus boundary layers

Boundary-layer control:

Transport restricted by diffusion across marginally stable BLs:

$$\tilde{Ra}^{(\delta)} \sim \tilde{Ra}_c$$

$$\frac{\delta}{H} \sim \tilde{Ra}^{-3}$$

$$\boxed{Nu \sim Ra^3 E^4}$$

« classical » regime of rotating convection.

Bulk control:

Transport restricted by convection through turbulent bulk:

$$Nu = \mathcal{F}(Ra\, E^{4/3}, Pr)$$

ν and κ play no roles.

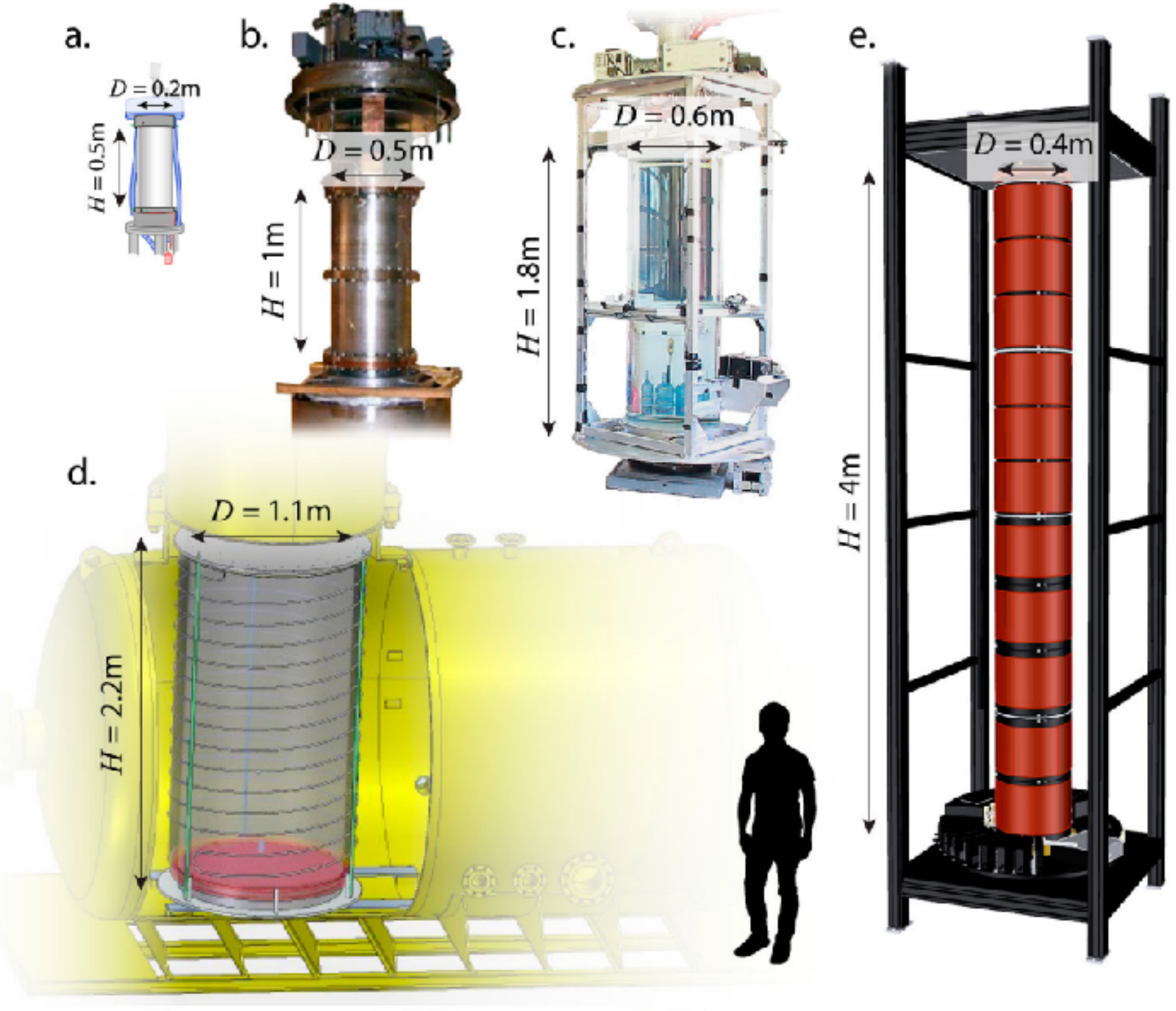
$$\boxed{Nu \sim Ra^{3/2} E^2 Pr^{1/2}}$$

Geostrophic Turbulence regime

Good news: the bulk should dominate over the BLs in the rapidly rotating regime!

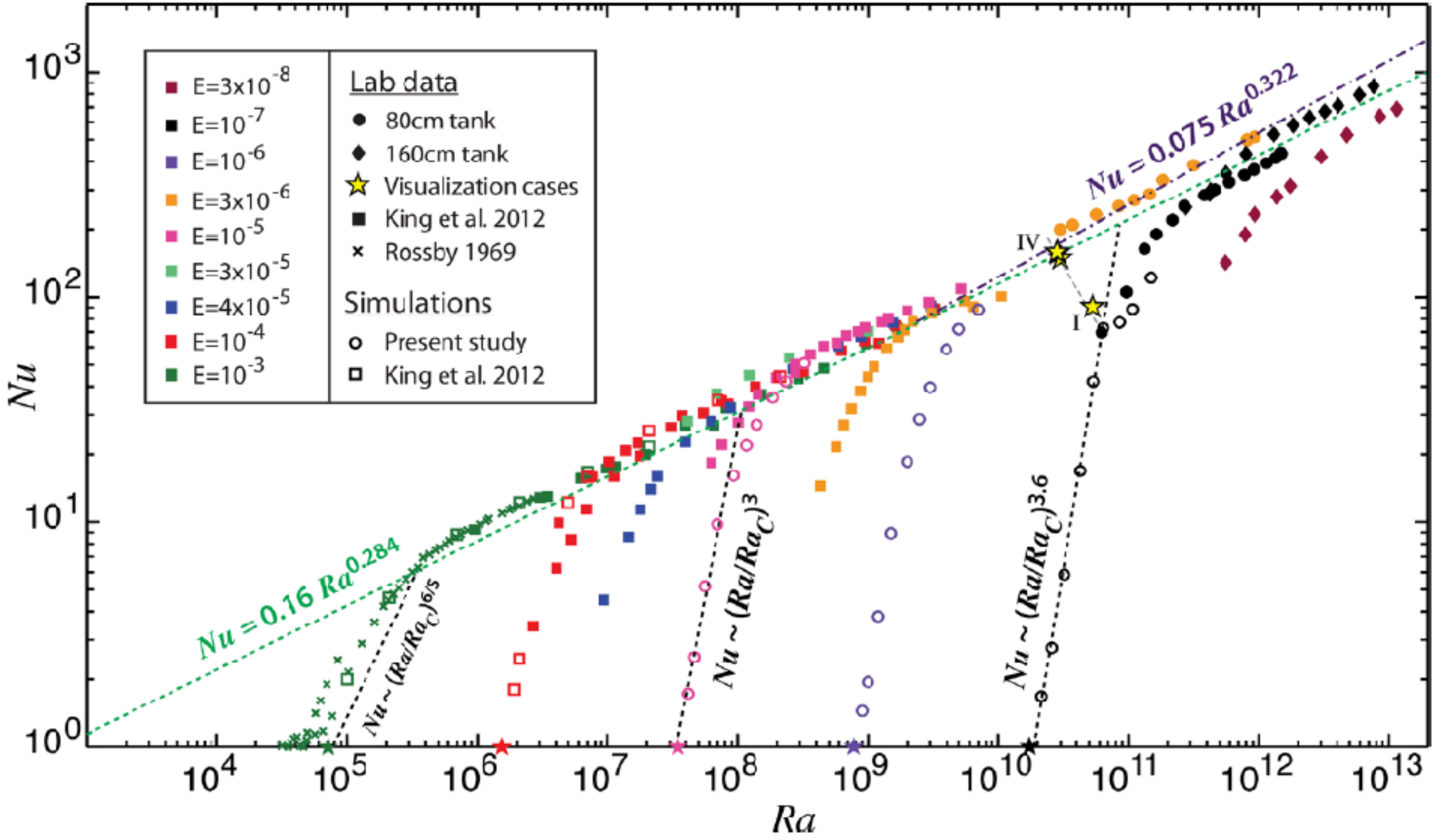
Ever-taller rotating RB cells

- Tall cells to reach low Ekman numbers.
- Avoid centrifugal effects: cigar shape.
- But then: boundary modes may pollute heat transport in the nonlinear regime.



[figure from Cheng et al. 2015]

Typical observations



Controlled by the boundary layers: no experimental observations of the geostrophic turbulence regime.

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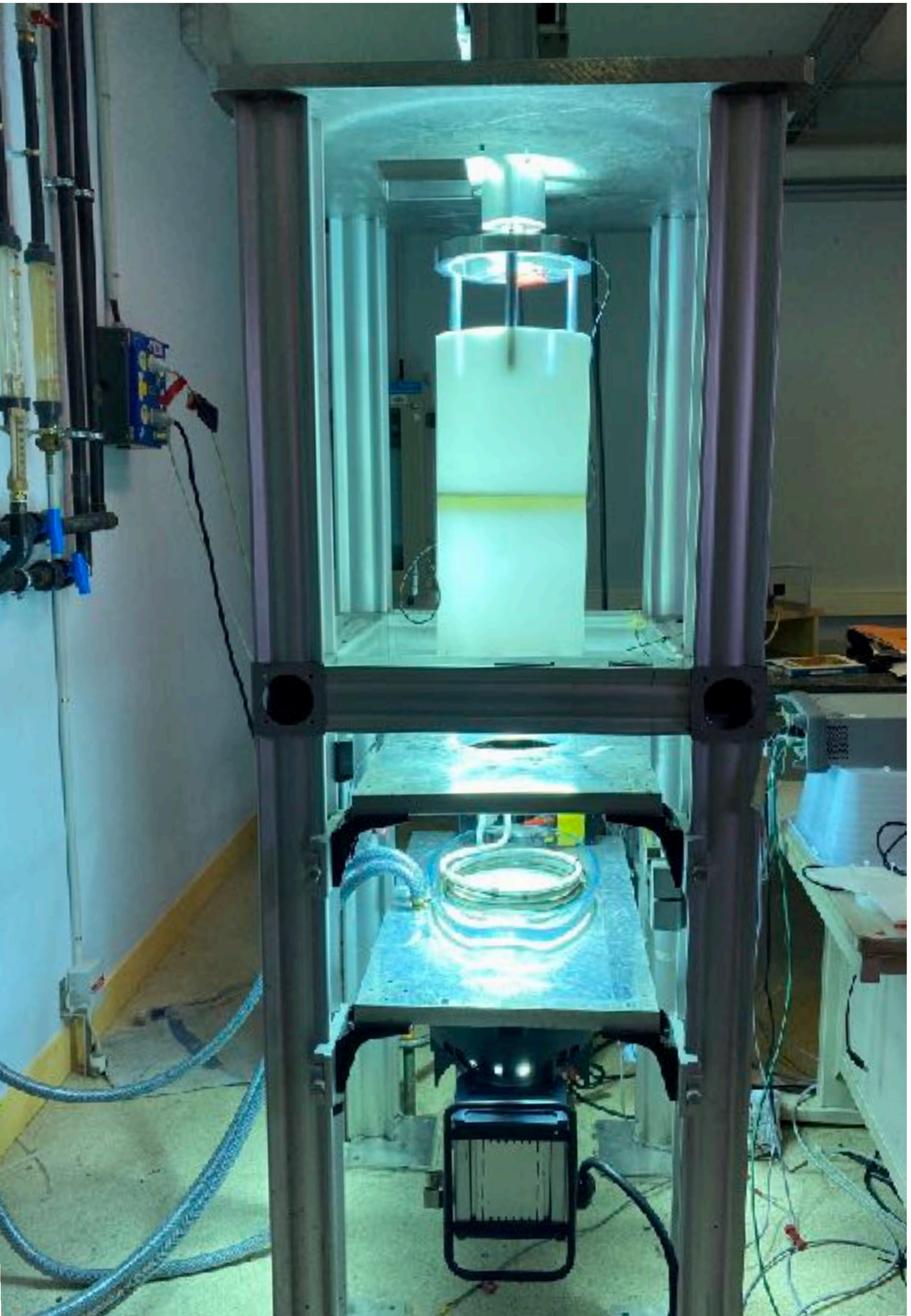
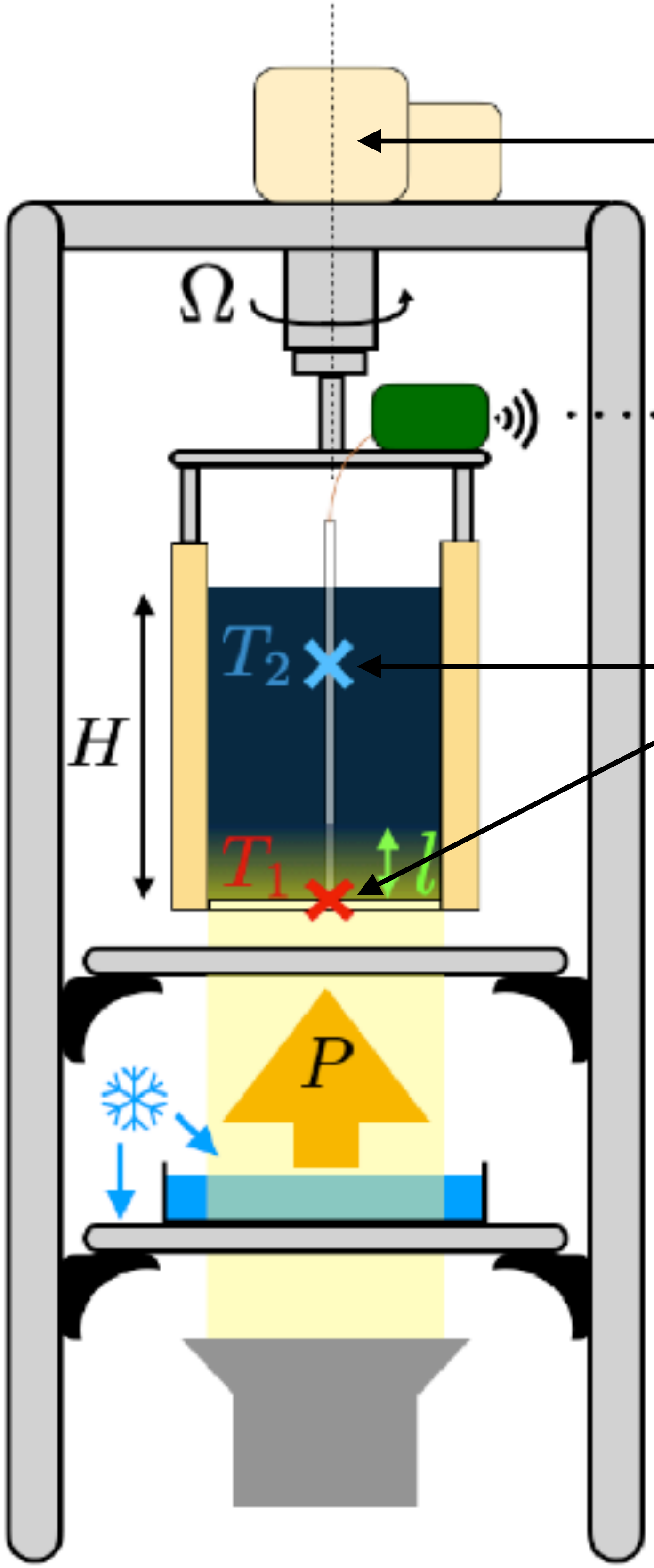
Radiatively driven rotating convection



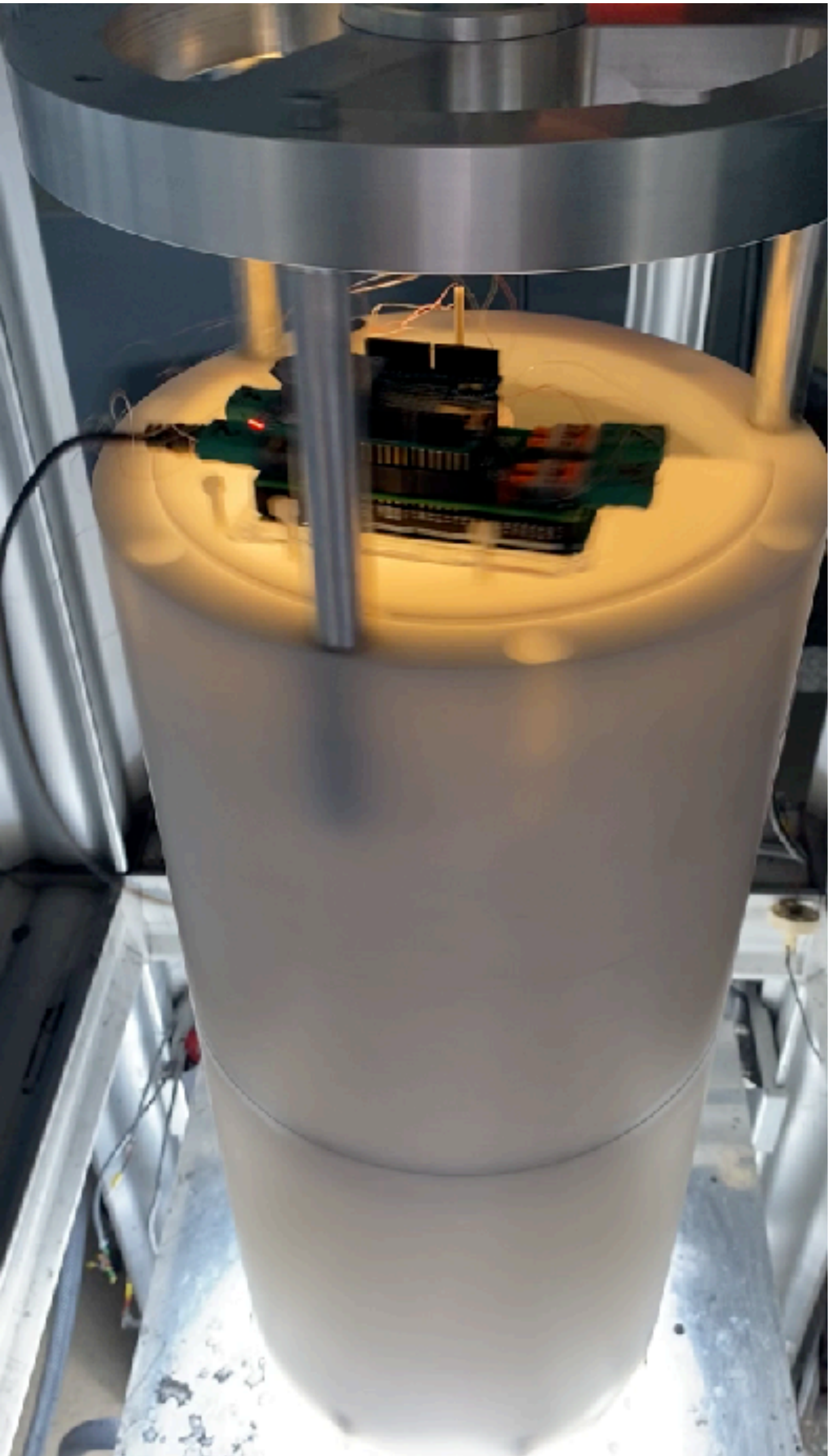


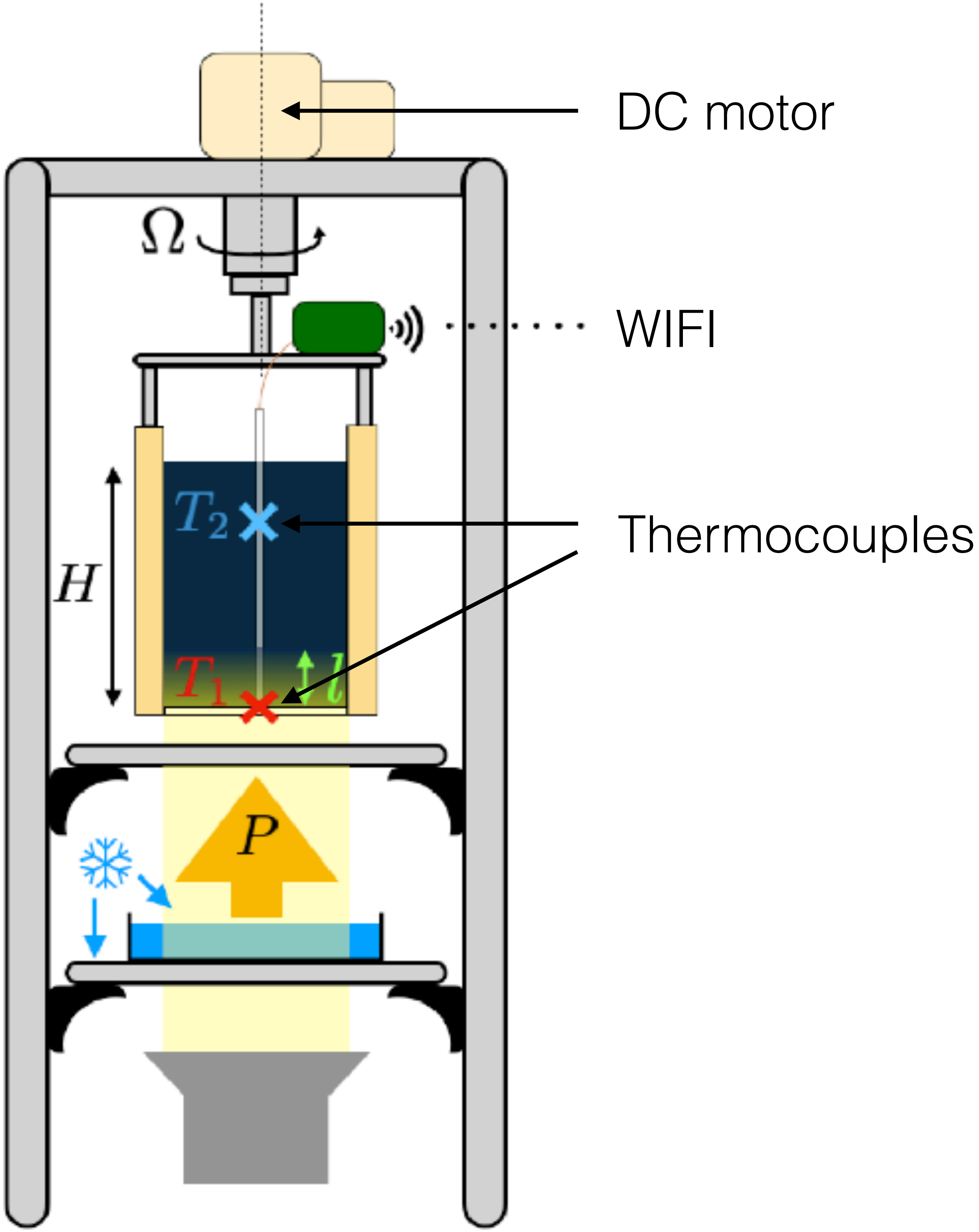
DC motor

WIFI

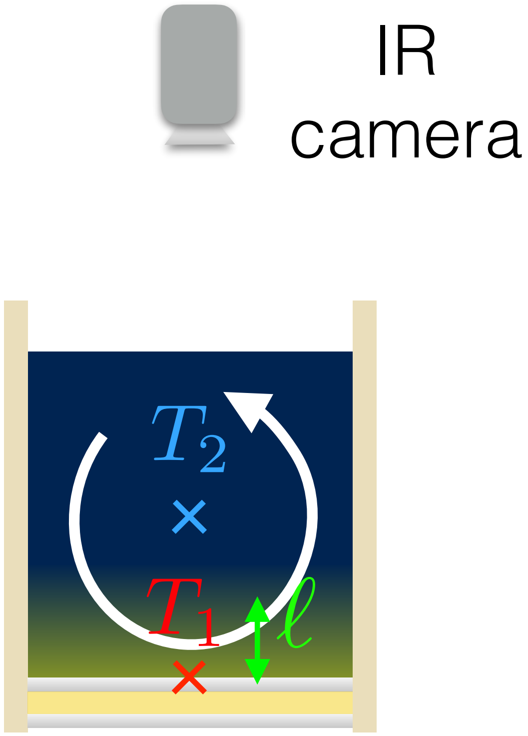
Thermocouples

Radiatively driven rotating convection



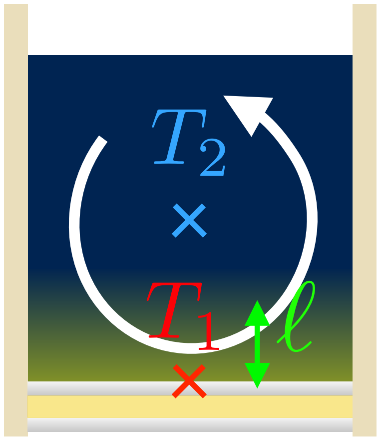


Impact of rotation: Qualitatively



Impact of rotation: Qualitatively

IR camera

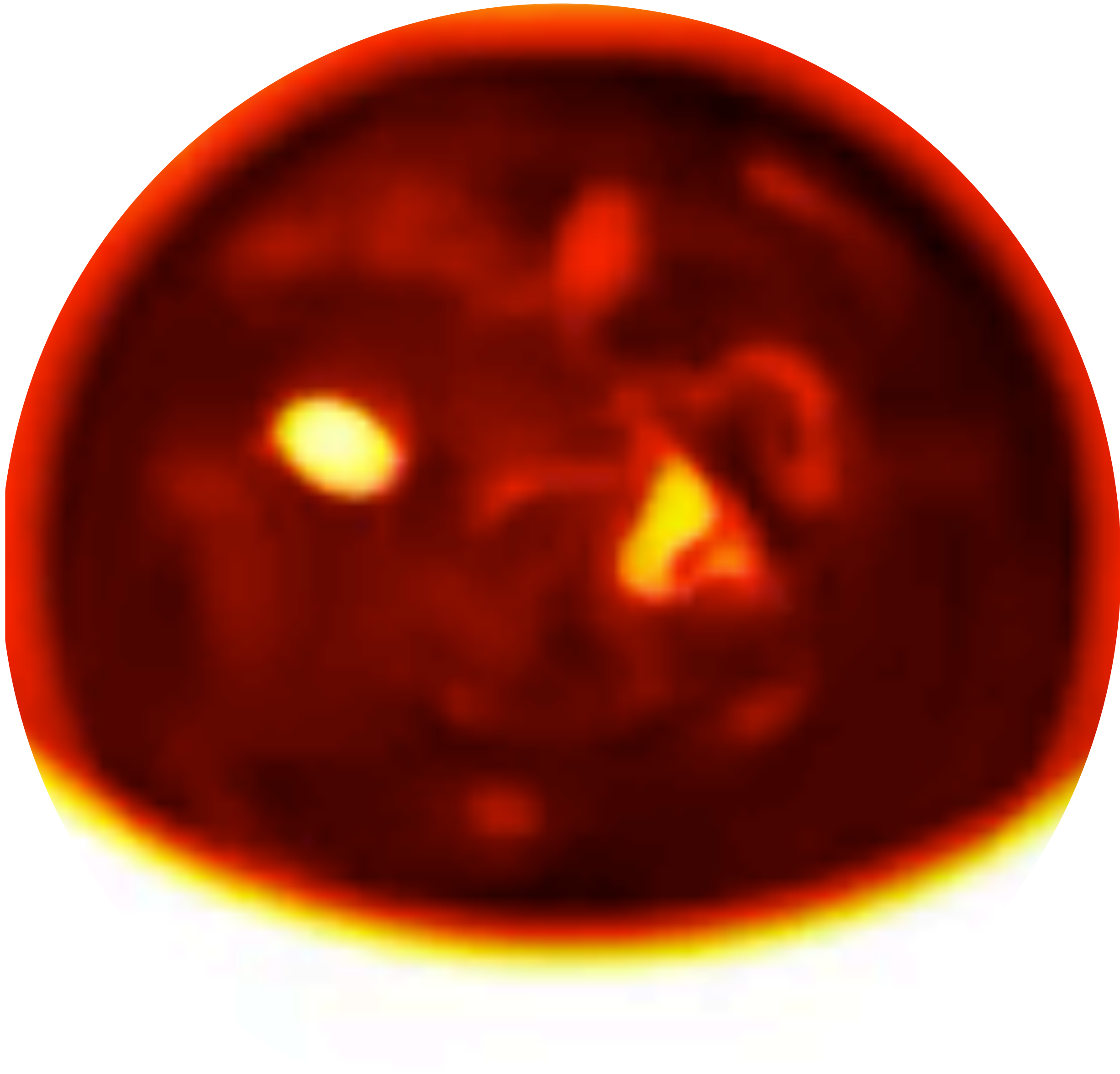


$$\Omega = 0$$

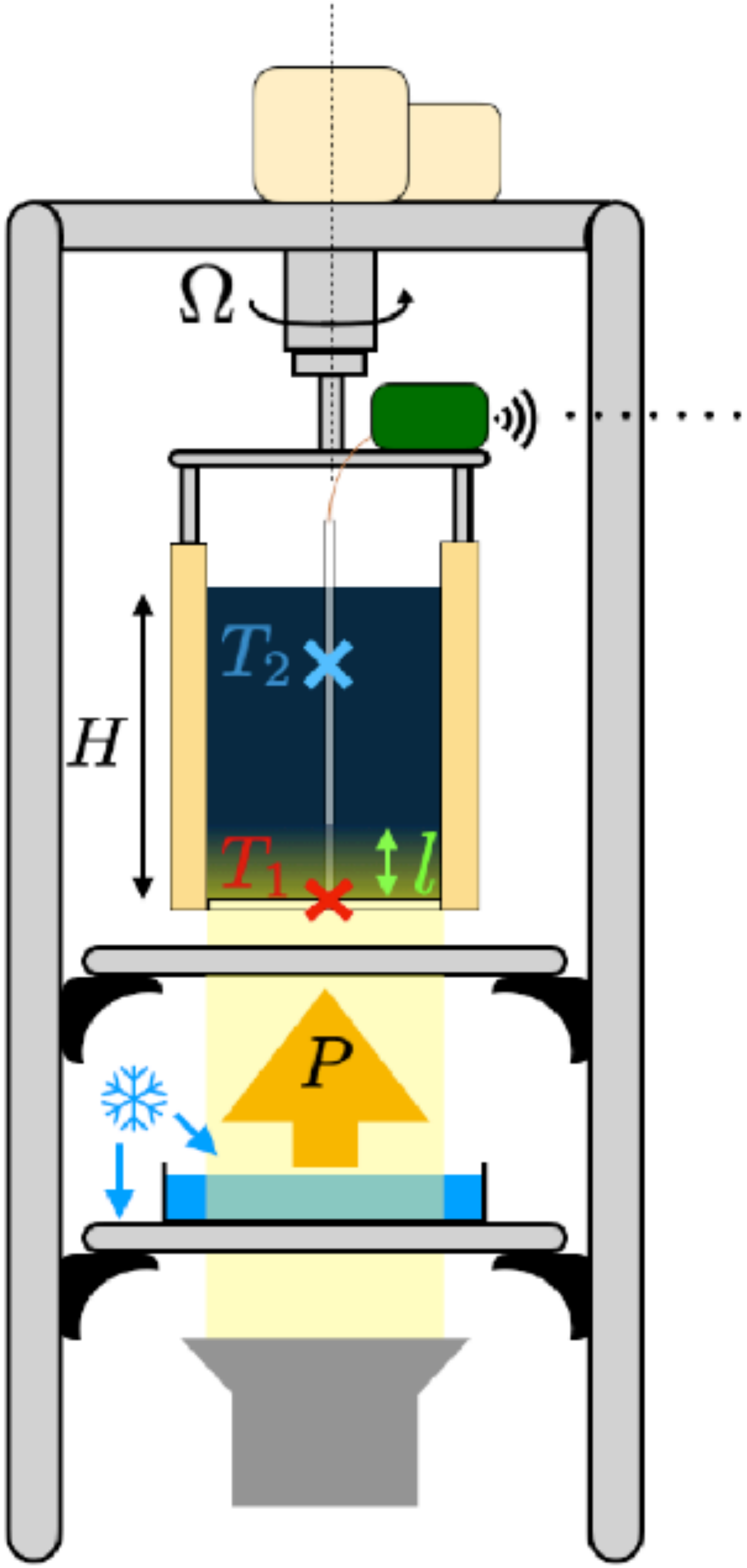
Impact of rotation: Qualitatively

$\Omega = 0$

$\Omega \neq 0$



Impact of rotation: quantitatively



- Heat flux is imposed:

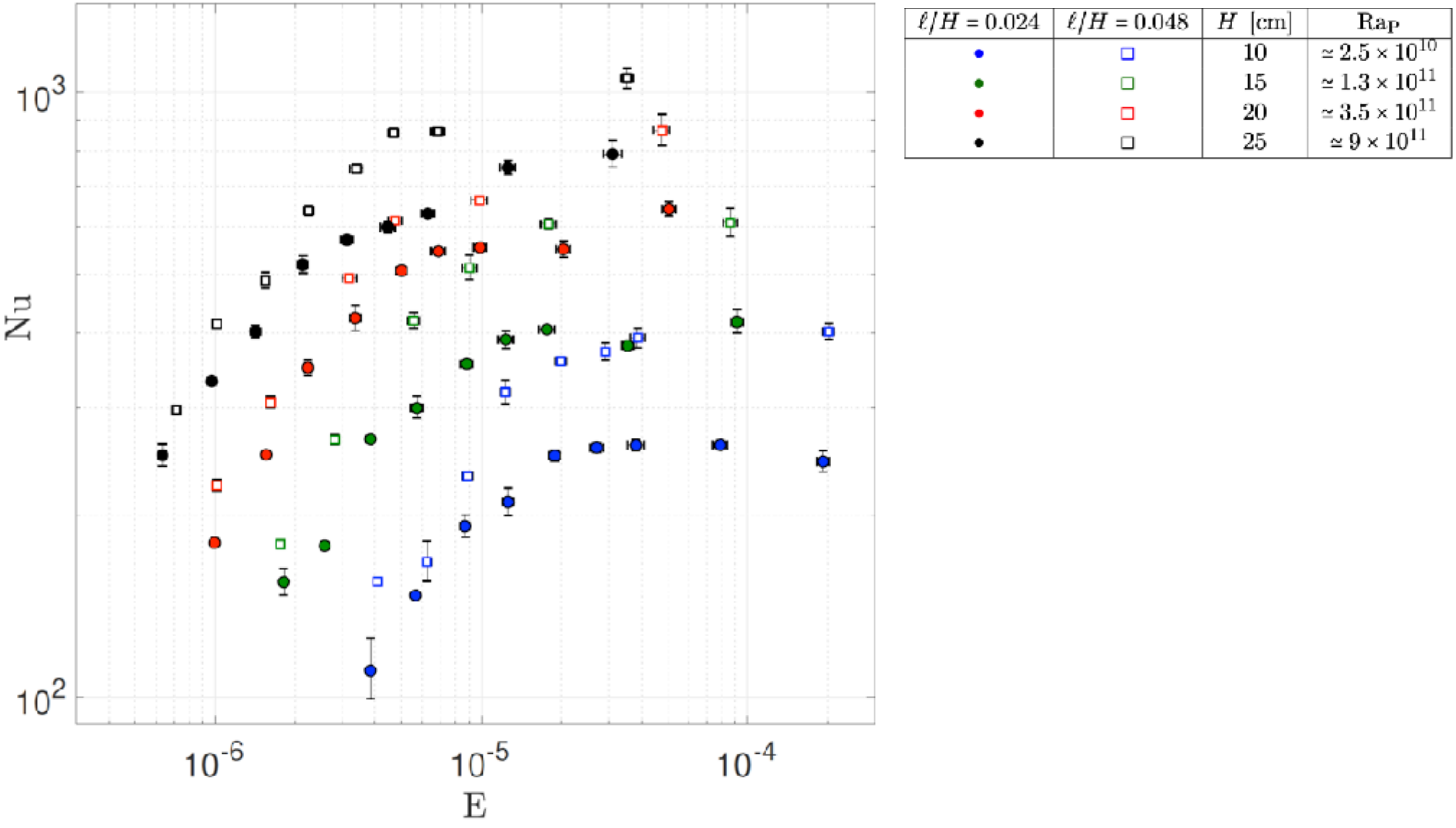
$$Ra_P = \frac{\alpha g P H^4}{\lambda \kappa \nu} = Nu \times Ra$$

- We measure the temperature drop ΔT to access Nu (and Ra if needed).

- Geostrophic turbulence scaling-law in terms of Ra_P is:

$$Nu \sim Ra_P^{3/5} E^{4/5} Pr^{1/5}$$

Rotation suppresses heat transport



$\ell/H = 0.024$	$\ell/H = 0.048$	H [cm]	Rap
●	□	10	$\simeq 2.5 \times 10^{10}$
●	□	15	$\simeq 1.3 \times 10^{11}$
●	□	20	$\simeq 3.5 \times 10^{11}$
●	□	25	$\simeq 9 \times 10^{11}$

Fully turbulent heat transport?

- Introduce diffusivity-free parameters to assess independence of molecular diffusivities ν and κ:

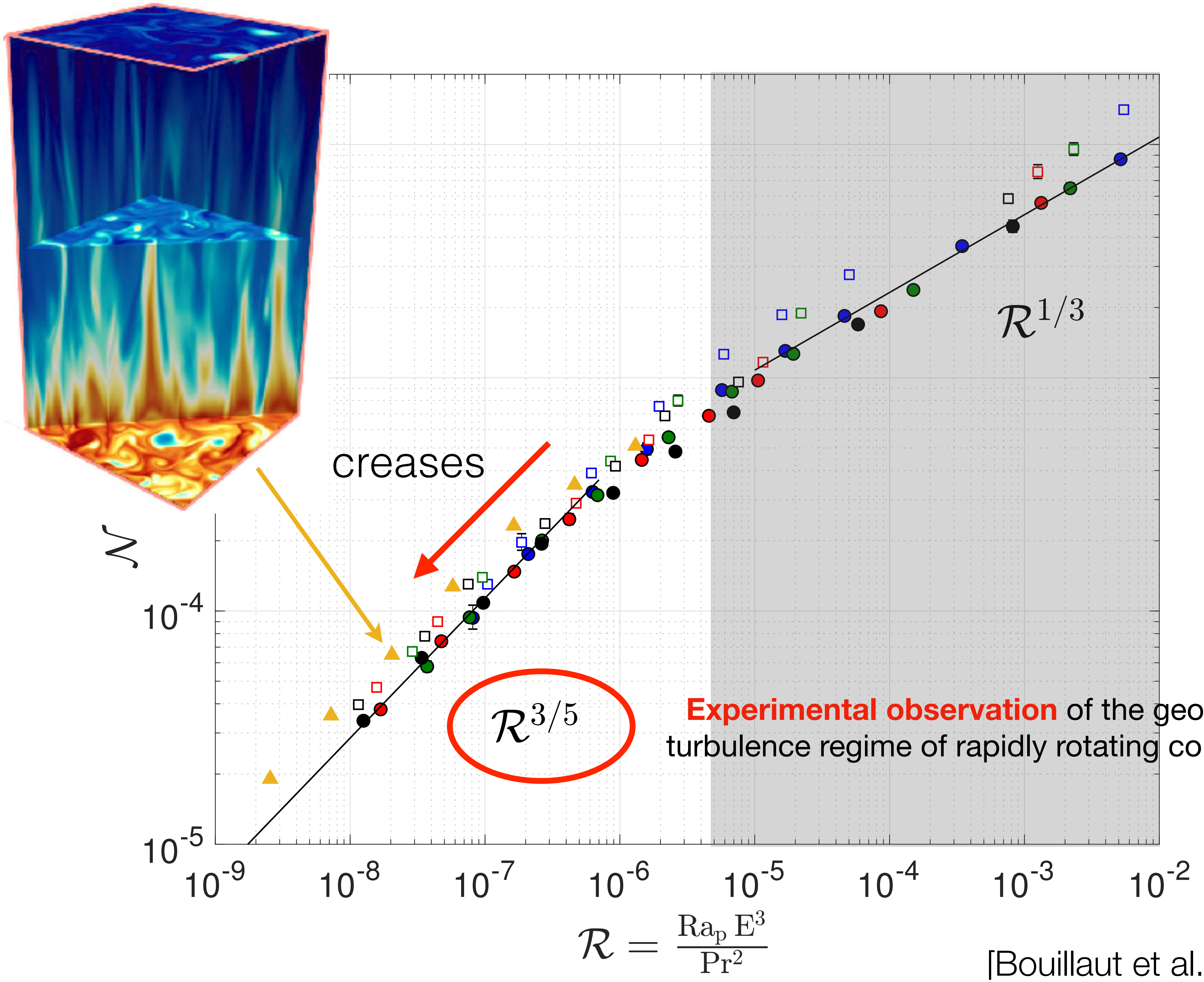
$$\mathcal{N} = \frac{Nu\,E}{Pr} \quad \text{vs.} \quad \mathcal{R} = \frac{Ra_P E^3}{Pr^2}$$

- All the data should collapse onto a master curve $\mathcal{N} = \mathcal{F}(\mathcal{R})$
  If heat transport is fully turbulent (« ultimate » regime).

- Geostrophic turbulence regime states that this master curve should be of the form:

$$\boxed{\mathcal{N} \sim \mathcal{R}^{3/5}}$$

Geostrophic turbulence regime



Experimental observation of the geostrophic turbulence regime of rapidly rotating convection !

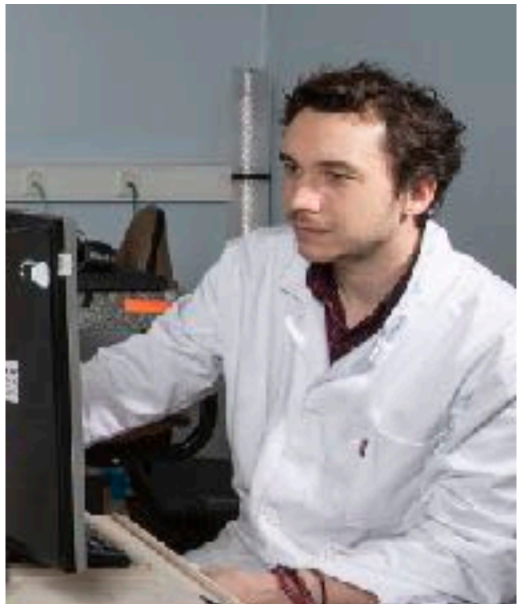
[Bouillaut et al., PNAS 2021]

Perspectives

The experiment can now be used to address several research questions:

- Can we validate the other scaling-laws associated with the geostrophic turbulence regime (velocity, temperature fluctuations, etc…)
- What is the structure of the temperature profile?
- How does it depend on the structure of the heat source (absorption length)?
- Ongoing debate on whether stars truly are in the rapidly rotating regime: what happens in the transitional region between the two « ultimate » regimes?

Thanks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[V. Bouillaut et al., PNAS, 2021]
[S. Lepot, S. Aumaître, BG, PNAS, 2018]
[V. Bouillaut, S. Lepot, S. Aumaître, BG, JFM, 2019]
[B. Miquel, S. Lepot, V. Bouillaut, BG, PRF, 2019]
[B. Miquel, V. Bouillaut, S. Aumaître, BG, JFM, 2020]

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