Cascades and reconnection in interacting vortex filaments

Alain Pumir

Laboratoire de Physique, ENS Lyon, France

ENS DE LYON

Max-Planck Institute for Dynamics and Self-Organization, Göttingen, Germany

MAX-PLANCK-GESELLSCHAFT

Joint projects with:

Michael Brenner and Ryan McKeown, Shmuel Rubinstein, and Rodolfo Ostilla-Monico

HARVARD John A. Paulson School of Engineering and Applied Sciences

האוניברסיטה העברית בירושלים THE HEBREW UNIVERSITY OF JERUSALEM

UNIVERSITY of HOUSTON

A fundamental law of turbulent flows

Turbulence occurs in flows at large Reynolds numbers.

$$Re \sim \mathcal{O}[(u\nabla)u]/\mathcal{O}[\nu\nabla^2 u]$$

$Re >> 1$ => "weak" dissipation in the fluid ??

Observation: in turbulent fluids, the energy dissipation,

$\nu\langle(\nabla^2 u).u\rangle = \nu\langle(\nabla u)^2\rangle$ is **finite**, independent of ν when $\nu \to 0$.

-- the ***dissipative anomaly***.

=> formation of **very strong velocity gradients**, the more so as $\nu \to 0$.

How does that happen ??

…an old question…

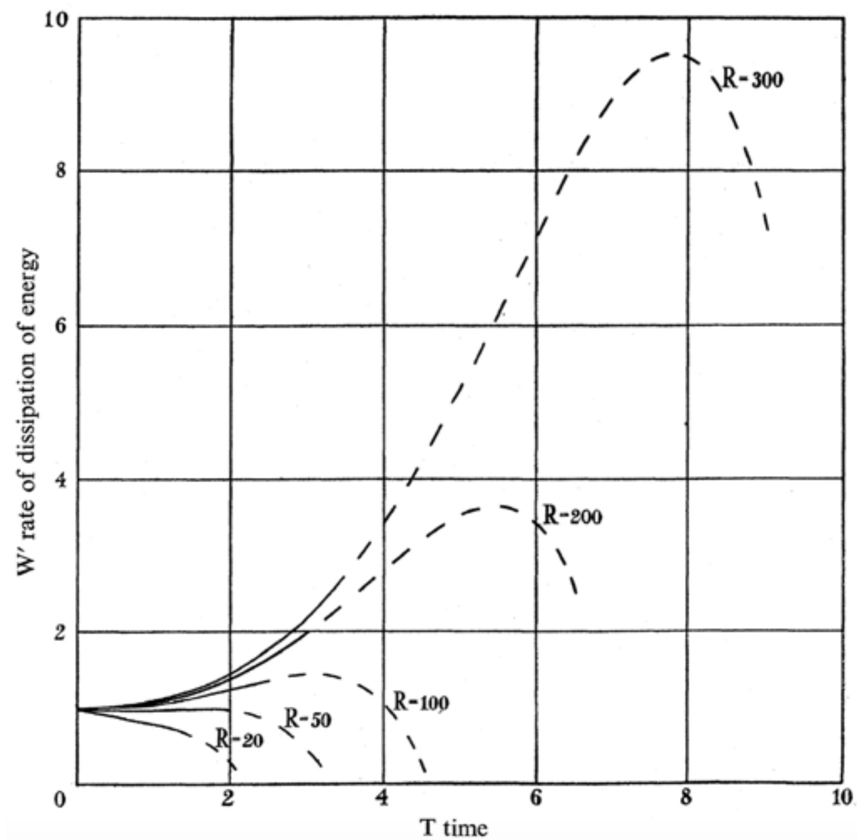
Mechanism of the Production of Small Eddies from Large Ones

By G. I. TAYLOR, F.R.S., and A. E. GREEN, B.A.

(*Received* 12 *October*, 1936)

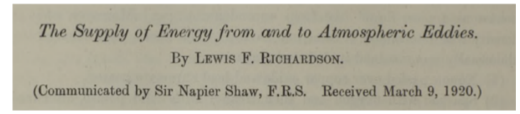
It is difficult to express these ideas in a mathematical form without assuming some definite form for the disturbance, but it is almost impossible to suggest an initial form which has the characteristics of the statistical isotropic turbulent motion to which (5) and (6) apply. Accordingly, we have searched for types of initial motion which have a definite scale and also have some of the properties of statistically uniform isotropic turbulence with a view to tracing the subsequent motion and finding out whether anything analogous to the process of the grinding down into smaller and smaller eddies occurs.

$$u = A \cos ax \sin by \sin cz$$
$$v = B \sin ax \cos by \sin cz$$
$$w = C \sin ax \sin by \cos cz$$



Taylor, G. I., and A. E. Green. "Mechanism of the production of small eddies from large ones. " Proceedings of the Royal Society of London. 158.895 (1937): 499-521.

The Turbulent Cascade


The Supply of Energy from and to Atmospheric Eddies.
By LEWIS F. RICHARDSON.
(Communicated by Sir Napier Shaw, F.R.S. Received March 9, 1920.)

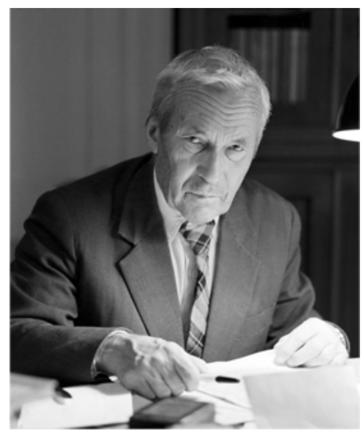
Big whorls have little whorls

That feed on their velocity

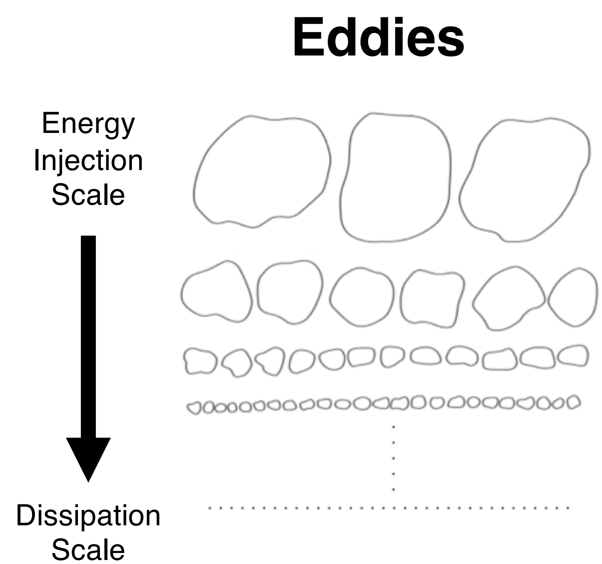
And little whorls have lesser whorls

And so on to viscosity

- L. F. Richardson



A. Kolmogorov



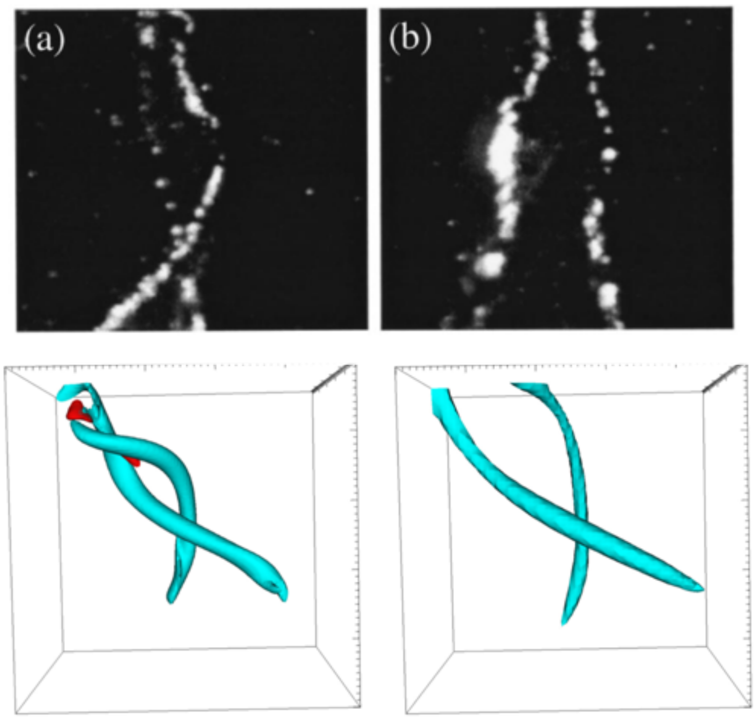
Questions

What is hiding behind the simple picture of a cascade ?

How does it work from a fluid mechanical point of view ?

Elementary turbulent structures have often been proposed in this context.
Point of view in this talk: what can one learn from
interacting vortex tubes ?

Why vortex tubes ?



(c) $(50\eta)^3$; C=500 (d) $(25\eta)^3$; C=1500

Visualization of the vortices through cavitation bubbles

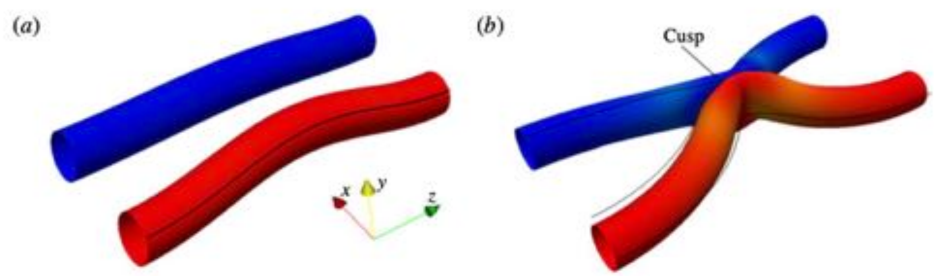
LaPorta et al, PoF 2000

Blown-up view of the most intense velocity gradient region.

Buaria et al, NJP 2019

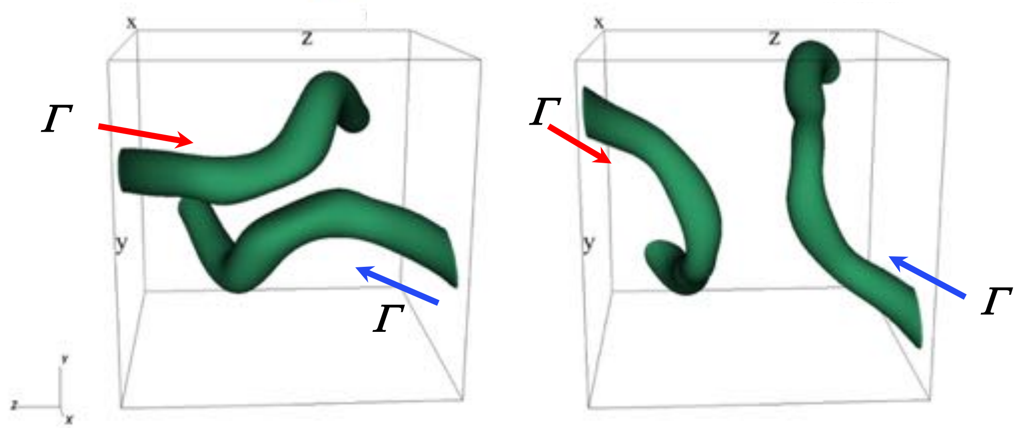
Reconnection in fluid mechanics

Consider the interaction between two interacting vortices.
One canonical configuration:



Yao and Hussain,
Annu. Rev. Fluid Mech, 2022

Observation:
Reconnection of tubes



Reconnection as a "singular mechanism" in fluid mechanics

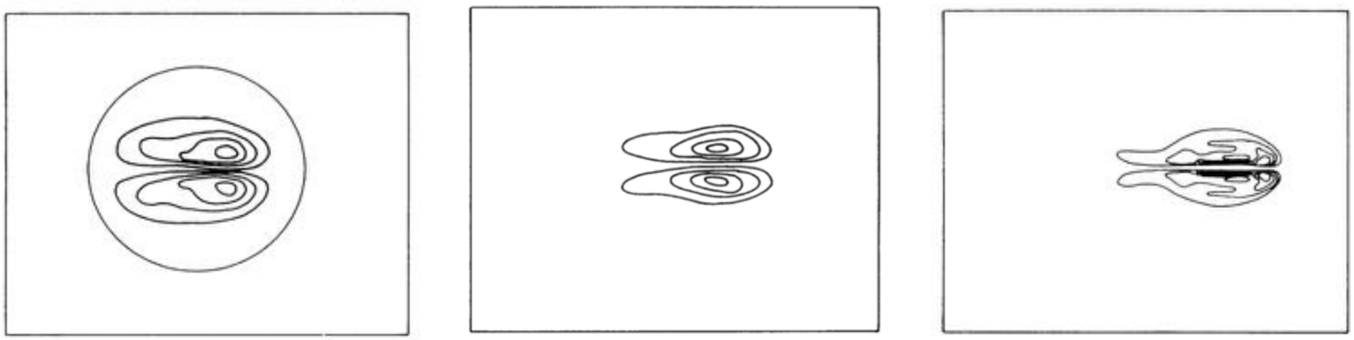
The process is 'singular' in the sense that reconnection involves a change of topology of vortex lines, ``forbidden'' in the limit $\nu \to 0$ $(Re \to \infty)$.

This does not seem to imply that the inviscid/infinite Reynolds number limit of the Navier-Stokes equations (Euler) is singular.

~ Unclear whether vorticity diverges during this process, even in the absence of viscosity, in the Euler equations (*Pumir + Siggia 1990, Kerr 1993, Hou 2007).*

Sheet formation

While studying the "canonical" configuration, one observes the ***formation of vortex sheets***, preceding reconnection (symmetry plane, z = 0):

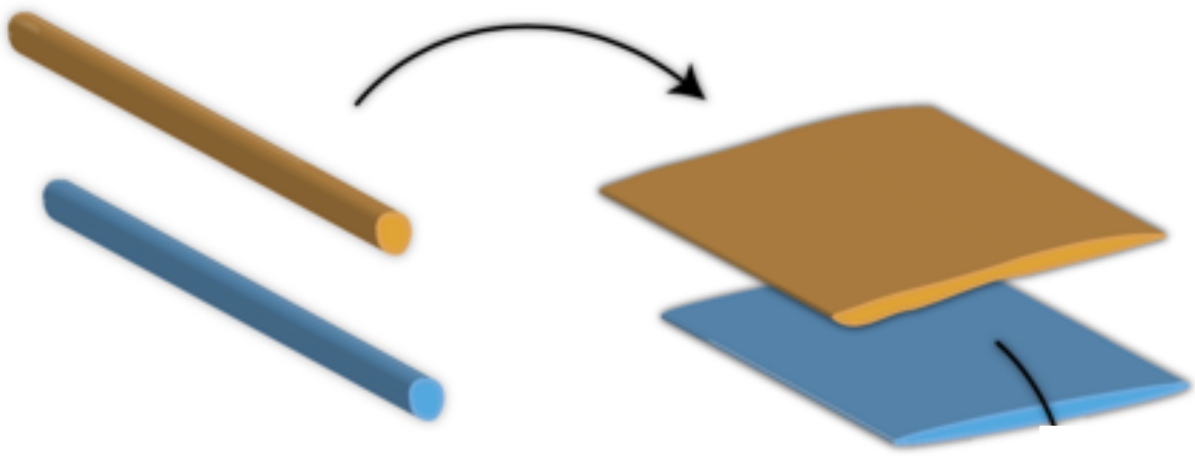


Pumir and Kerr, PRL 1987
[and many more…]

The presence of vortex sheets, which are quasi-2-dimensional structures, considerably slows down the growth of vorticity !

Interaction of vortex tubes

The interaction of antiparallel vortex tubes is an ideal problem to study vorticity amplification (Siggia, 1985, Siggia +AP, 1985 & 87, Brenner and Hormoz, 2012, etc…)



Consequence:
amplification of vorticity gradients is considerably hindered…

Could singularities proceed past the formation of quasi-2d structures?

Quasi 2d structure: could undergo instabilities, restoring the 3d-character of the flow, hence the stretching.

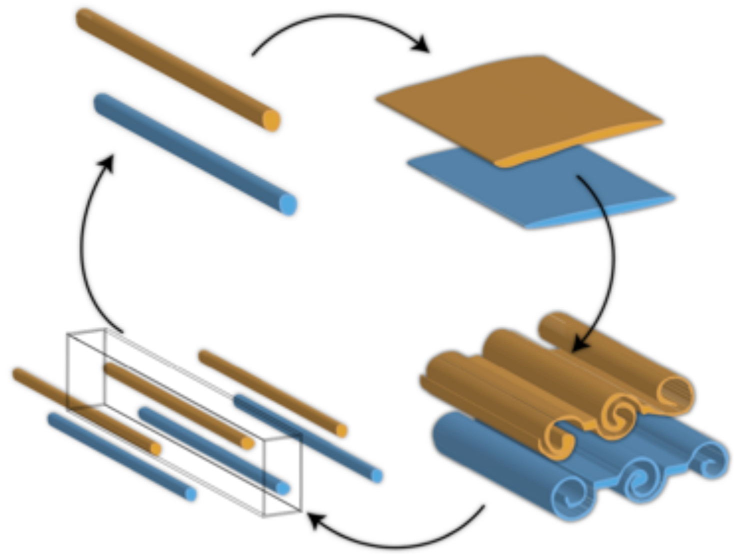


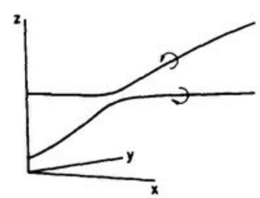
FIG. 1. Schematic of the proposed mechanism for iterated vortex interactions. Initially two antiparallel vortex filaments collide, resulting in the formation of two finite-thickness antiparallel vortex sheets. These sheets then destabilize, resulting in two arrays of antiparallel vortex filaments, each of which has a smaller core size, circulation, and separation distance than the initial filaments. The newly formed antiparallel filaments then approach each other and the process continues. This paper carries out scaling estimates based on similarity solutions of the Biot-Savart equations and investigates the plausibility of this scenario.

Brenner, Hormoz and Pumir, 2016
See also Terry Tao, 2014

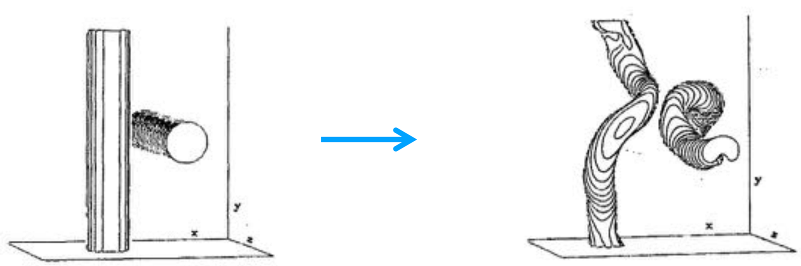
Universality in reconnection ?

"*The overall reconnection dynamics should not depend on the initial spatial configuration*" (Yao and Hussain, Annu. Rev. Fluid Mech. 2022).

✓ The Biot-Savart dynamics brings together anti-parallel filaments (*Siggia, 1985*).



✓ Other initial conditions seem to lead to qualitatively similar results. Example: two perpendicular tubes *(Boratav et al, 1992*).



Cascade in the interaction of two antiparallel vortex tubes

Experiment (R. McKeown and S. Rubinstein): collision of two vortex rings (see also Lim and Nickels, Nature 1992).

Cascade in the interaction of two antiparallel vortex tubes

- Intense interaction between the vortices; little trace of the formation of vortex sheets seen during "reconnection".

- One can think of the interaction between the tubes rather in terms of an iterative cascade of instabilities, induced by the elliptic instability when two antiparallel tubes interact.

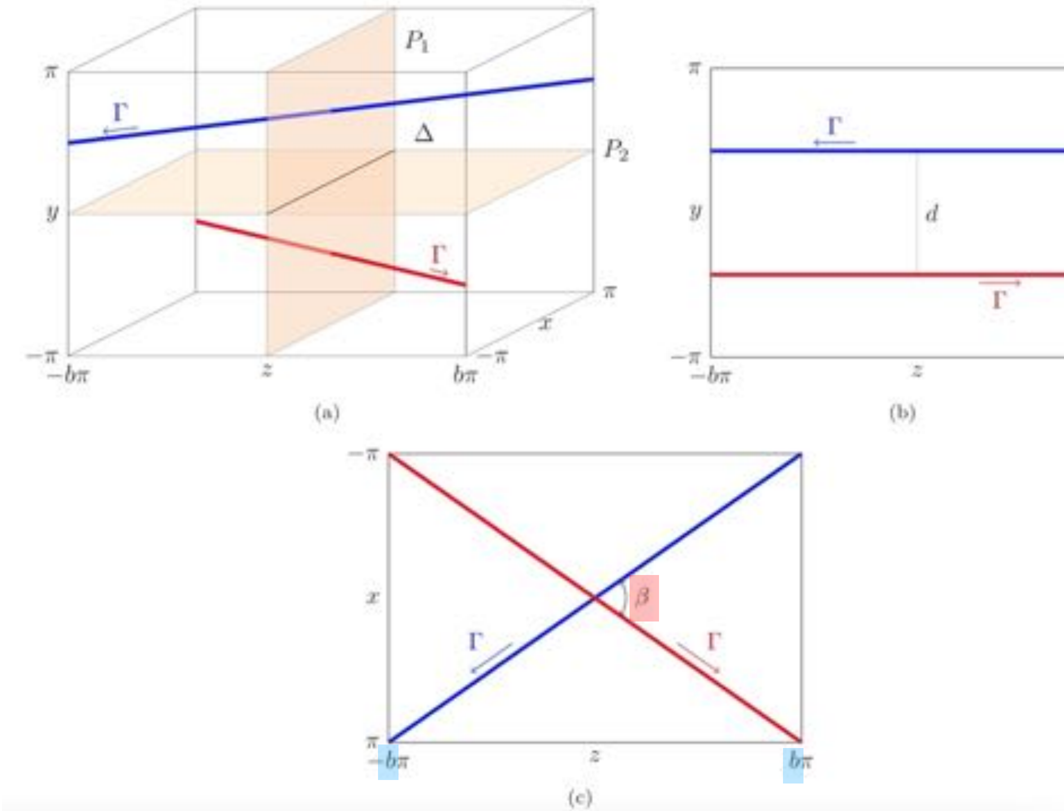
Spinning gyres meet antigyres,
Create sheets with instability,
Then new gyre pairs at angles right,
Spur endless repeatability.

(Alan Newell)

- Development of a turbulent, $E(k) \sim \varepsilon\, k^{-5/3}$ regime after a few steps of instability (McKeown et al, Sci. Advances 2020).

Two types of evolution when tubes interact ?

A parameteric study: two tubes initially at an angle



Lay two vortex tubes, whose centers are on the two straignt lines.

- Vary the aspect ratio of the box, b
  => vary the angle β.

n.b. $b = \text{cotan}(\beta/2)$

- Aspect ratio = 1: $\beta = 90^o$
  ~ same as Boratav 1992.

Numerical study of interacting vortex tubes

Runs	1	2	3	4	5	6	7	8	9	10	11	12A/B/C
b	1	1	1	1	1	5/4	3/2	2	5/2	3	4	∞
β	90°	90°	90°	90°	90°	77.3°	67.4°	53.1°	43.6°	36.8°	28.1°	0°
Re_Γ	2200	3300	4000	4550	5400	4000	4000	4000	4000	4000	4000	4000
N_l	256	256	384	384	384	192	240	192	192	192	192	192*
N_h	384	512	512	512	512	384	400	320	320	320	320	320*

$b = 1$; Re_Γ increases

b increases; $Re_\Gamma = 4000$

b "∞"; $Re_\Gamma = 4000$

Reconnection in the case β = 90°

$Re_{\Gamma} = \Gamma/\nu = 4000$

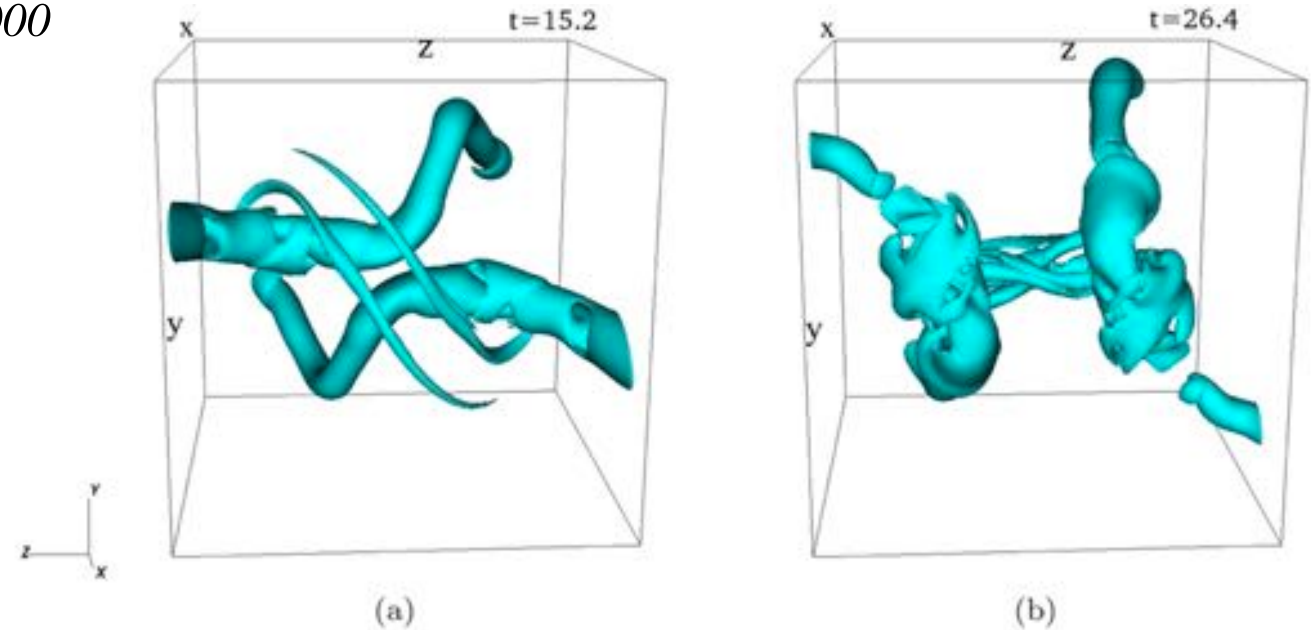
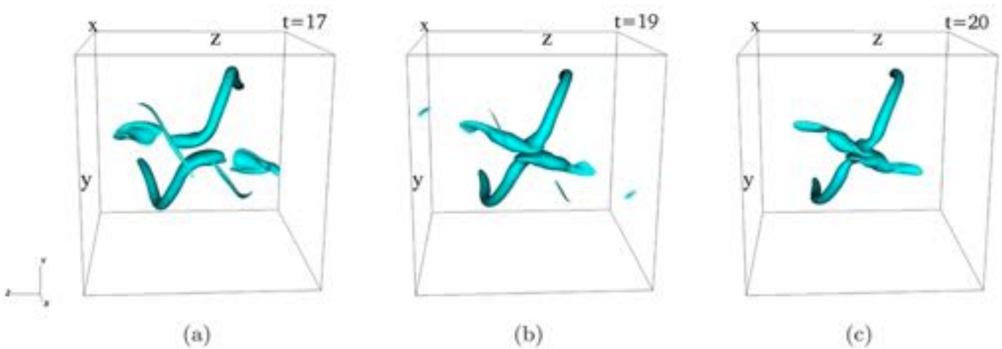
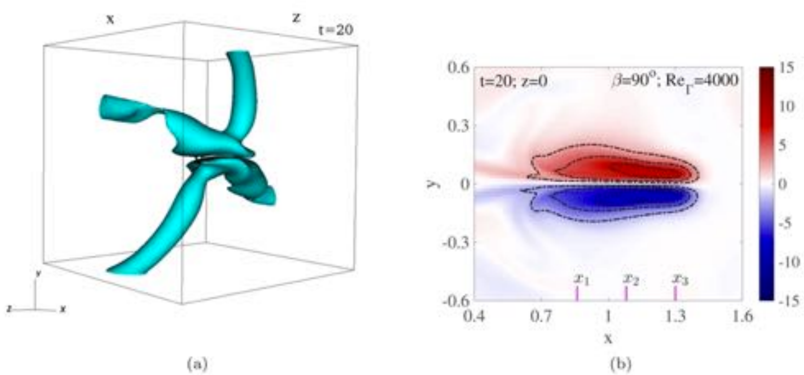


Illustration of the solution before the tubes interact (a) and when they have reconnected (b)

Reconnection in the case $\beta = 90^o$
Sheet formation and interaction

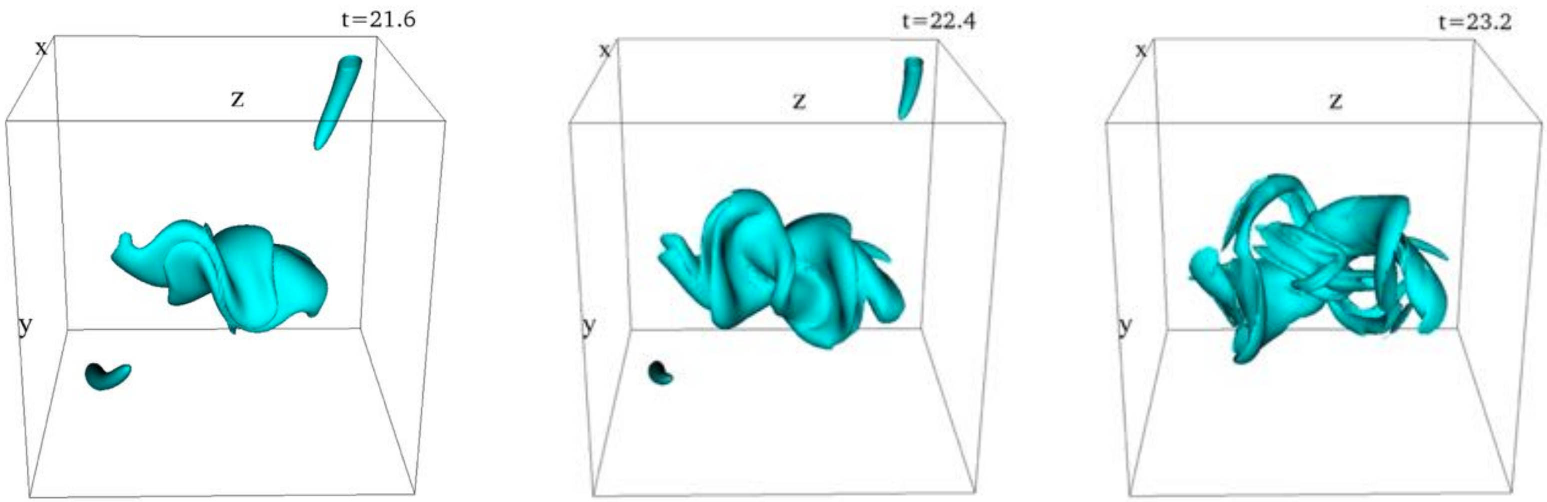


Upper panel: Formation of vortex sheets as the tubes come together



Lower panel: Detail of the sheet structure; Contour plot of vorticity in the symmetry plane.

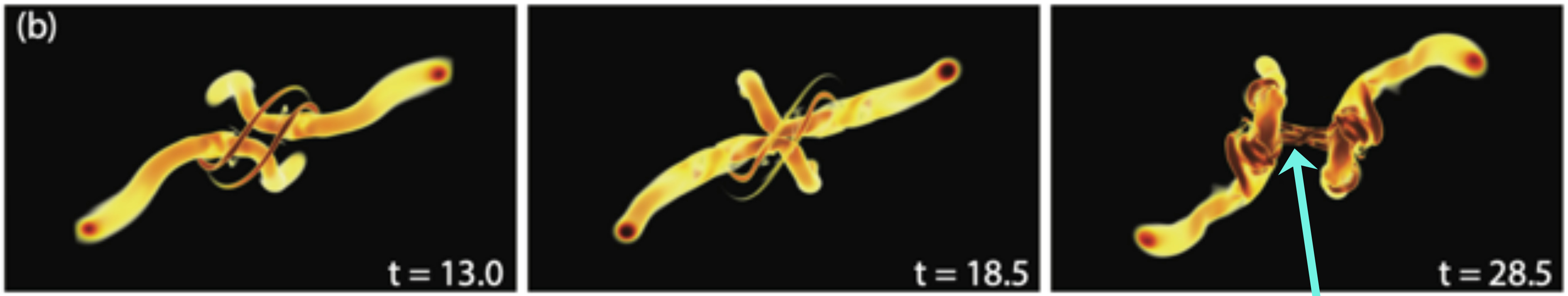
Reconnection in the case $\beta = 90^o$
Twisting of the sheets and reconnection



The vortex sheets twist each other, leaving behind a tangle of small size vortices, forming a bow behind the vortices.

=> Small scale structures in the back of the region that has reconnected.

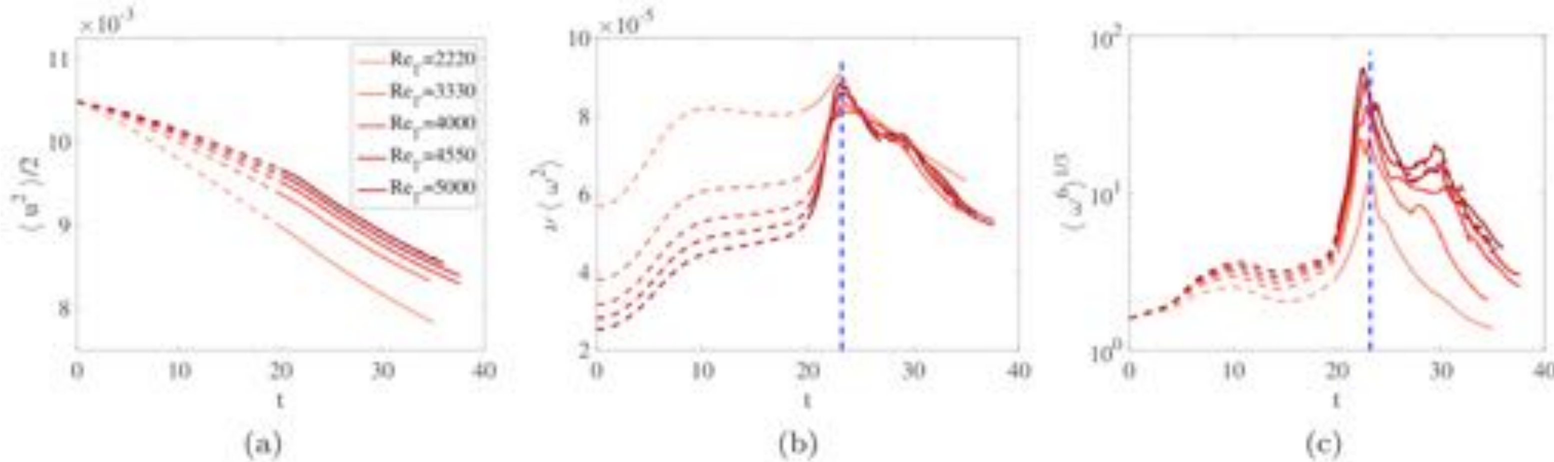
Small scale after reconnection



braids

As Re_{Γ} increases, smaller and smaller scales form in the braid. Cascade mechanism ?

Reconnection at $\beta = 90^o$: Reynolds number dependence



Kinetic energy vs. t

Dissipation vs. t

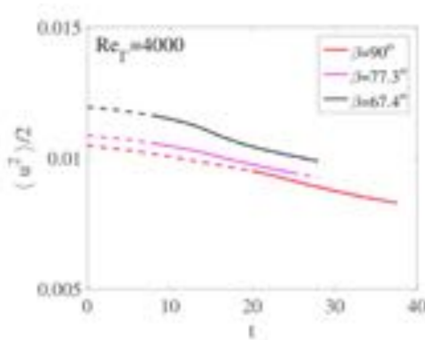
Higher (6th) moment of vorticity vs. t

- ✓ The reconnection time does not depend on Re_Γ; but
- ✓ larger gradients are created when Re_Γ increase.

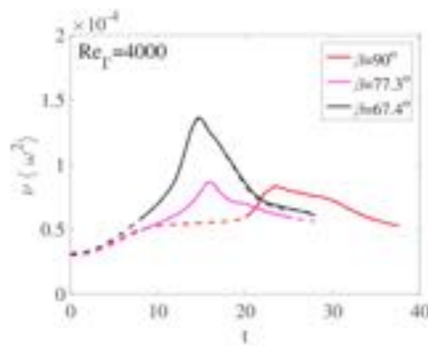
How robust is the scenario for $\beta = 90^o$?

- A qualitatively similar phenomenology is observed when $\beta = 77.4^o$ ($b = 5/4$).
- Qualitative deviations are observed already when $\beta = 67.4^o$ ($b = 3/2$).

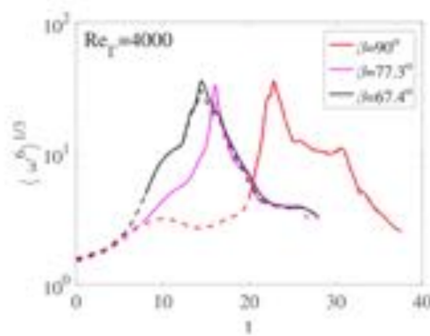
nb: quantitative differences when varying b (or β); consequences of the Biot-Savart dynamics.



Kinetic energy vs. t



Dissipation vs. t



6^{th} moment of vorticity vs. t

Reconnection in the case $\beta = 28.1^o$

$Re_\Gamma = \Gamma/\nu = 4000$

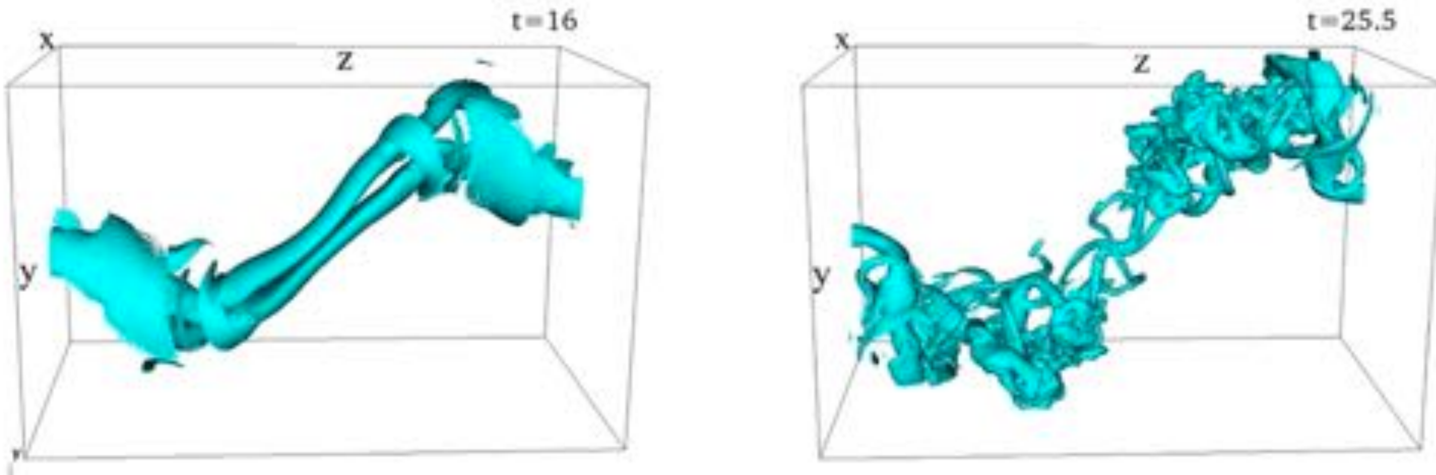
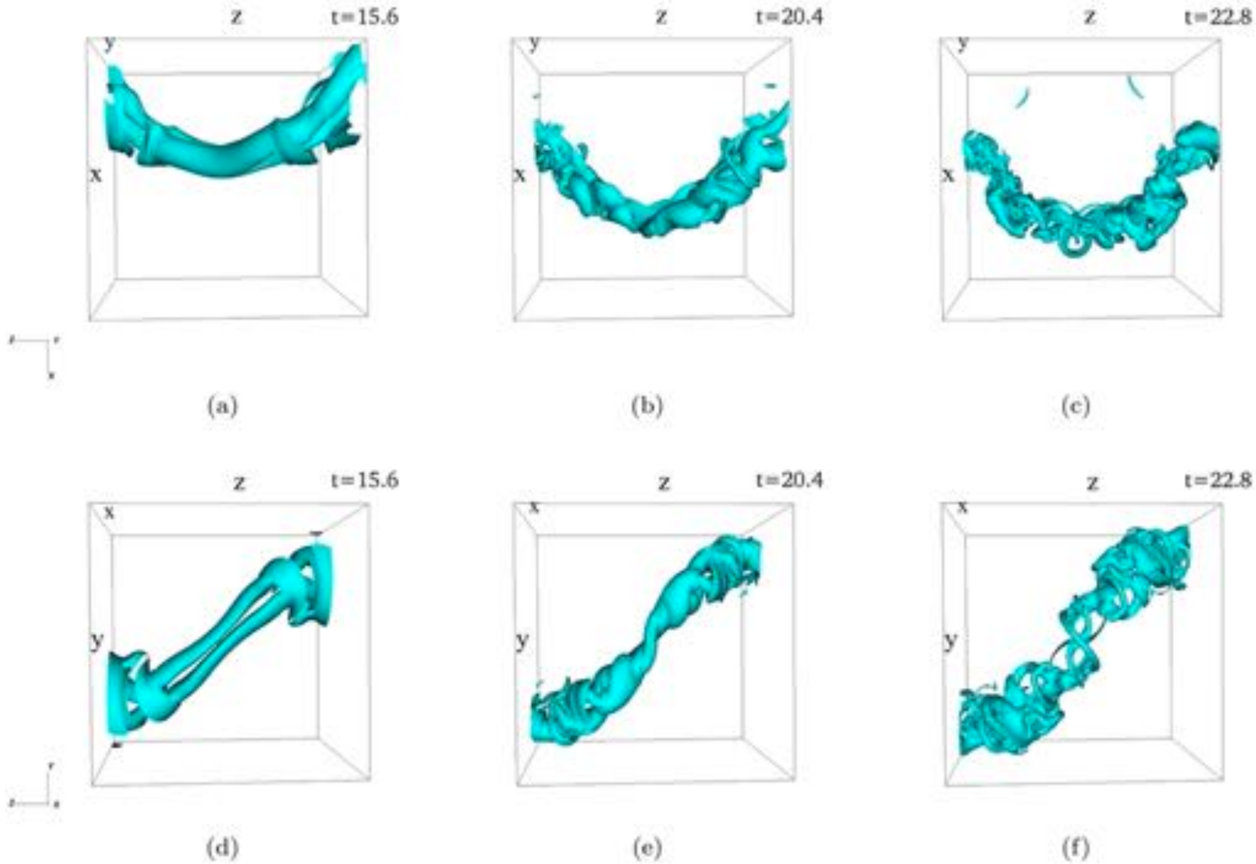


Illustration of the solution as the tubes begin to interact (a) and during the interaction (b)

Reconnection in the case $\beta = 28.1^o$

Top view

Font view

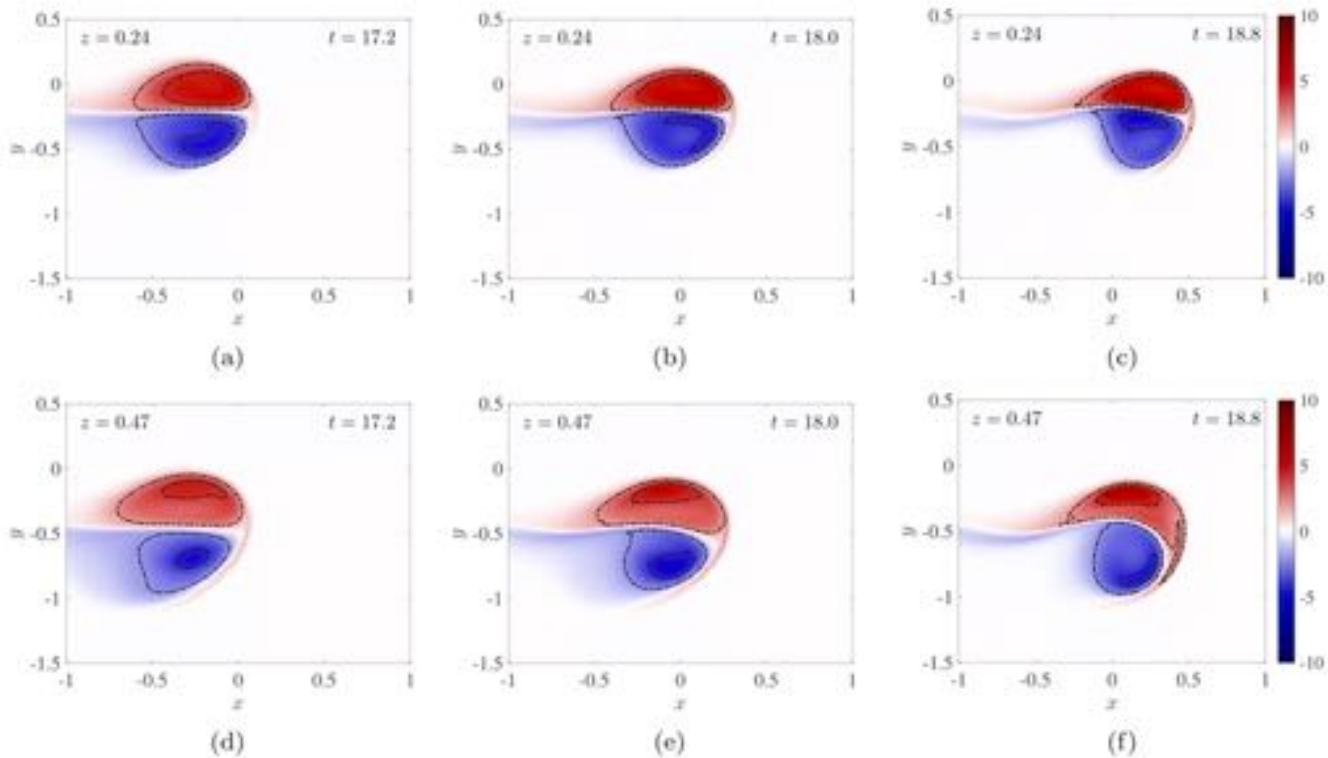


Evolution of the solution: instabilities of the core: “cascade of small scales” .

Reconnection in the case $\beta = 28.1^o$
Core structure

Top view

Font view



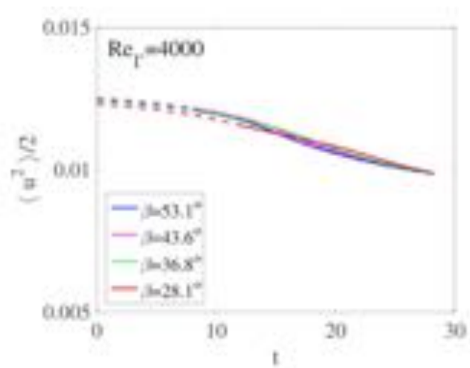
✓ No evidence of strong vortex sheets ! Reconnection proceeds very differently

✓ Strong role of the core in the dynamics !

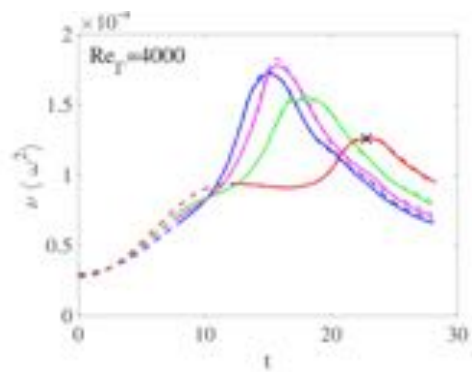
How robust is the scenario for $\beta \lesssim 67.5^o$?

- The absence of any sheet formation is observed for $b > 3/2$ ($\beta = 67.4^o$)_.
- Instead, instabilities are developing along the core, qualitatively similar as the one seen at $b = 4$ ($\beta = 28.1^o$).

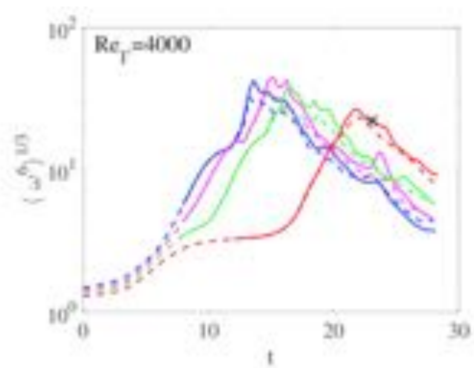
nb: As it was the case for $b \lesssim 3/2$, quantitative differences when varying b (or β); consequences of the Biot-Savart dynamics.



Kinetic energy vs. t



Dissipation vs. t



6^{th} moment of vorticity vs. t

Conclusions

- More than just one mechanism leading to reconnection of vortex tubes; crucially involves the core structure (*nothing to do with reconnection in superfluids !*).

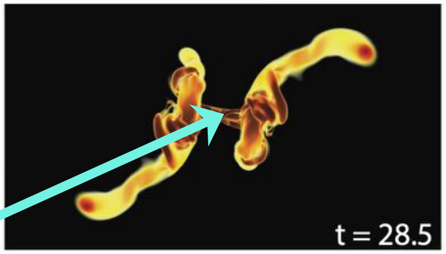
- Vortex sheets are not necessarily part of the interaction between two tubes.

- Cascade mechanisms:

✓ involve rather vortex cores and the formation of transverse tubes of vorticity ($b > 3/2$). Similarly in an iterative manner (as in McKeown et al, 2020)

✓ Very significant small scales is observed in the wake of the reconnecting tubes, after sheets have disappeared ($b < 3/2$).

Braids left behind after interaction ($b=1$; $\beta=90^o$).



Results of the cascade of instability (large b, $\beta \sim 0$).

Thank you for your attention !

Questions ?

References:
Brenner *et al*, Phys Rev Fluids, ***1**, 084503,* 2016.
McKeown *et al*, Phys. Rev. Fluids, ***3**, 124702*, 2018.
McKeown *et al*, Sci. Advances, aaz2717, 2020.
Ostilla-Monico *et al*., Phys. Rev. Fluids, ***6**, 074701,* 2021.
Mishra *et al*., Phys. Rev. Fluids,6*, 104702* (2021)