Atmospheres and their numerical treatments in astrophysics

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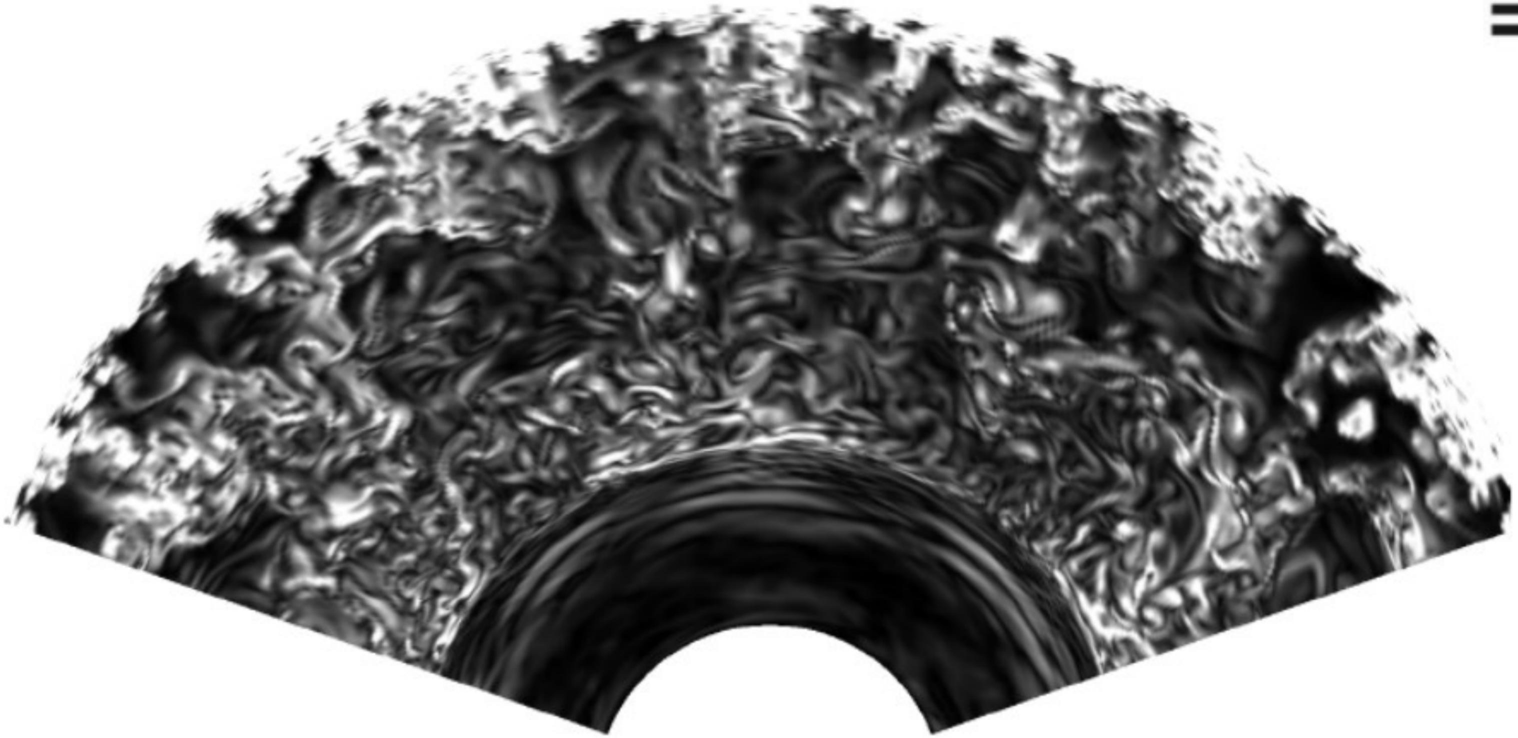


3D stellar turbulent convection (J. Pratt)

Acknowledgements: European Research Council TOFU PI: I. Baraffe

PSMN Pôle Scientifique de Modélisation Numérique

GENCI Grand Equipement National pour le Calcul Intensif

PNHE Programme national hautes énergies

Outline

1. Atmospheres : from planets to neutron stars and black-holes
2. Ordinary stars : insights and numerical challenges
3. A fully implicit scheme
4. A well-balanced scheme
5. The case of overshooting and lithium depletion on the sun
6. Conclusions

Earth from 45’000 km distance

Image taken by Apollo 17,
December 7, 1972,
image credit: NASA

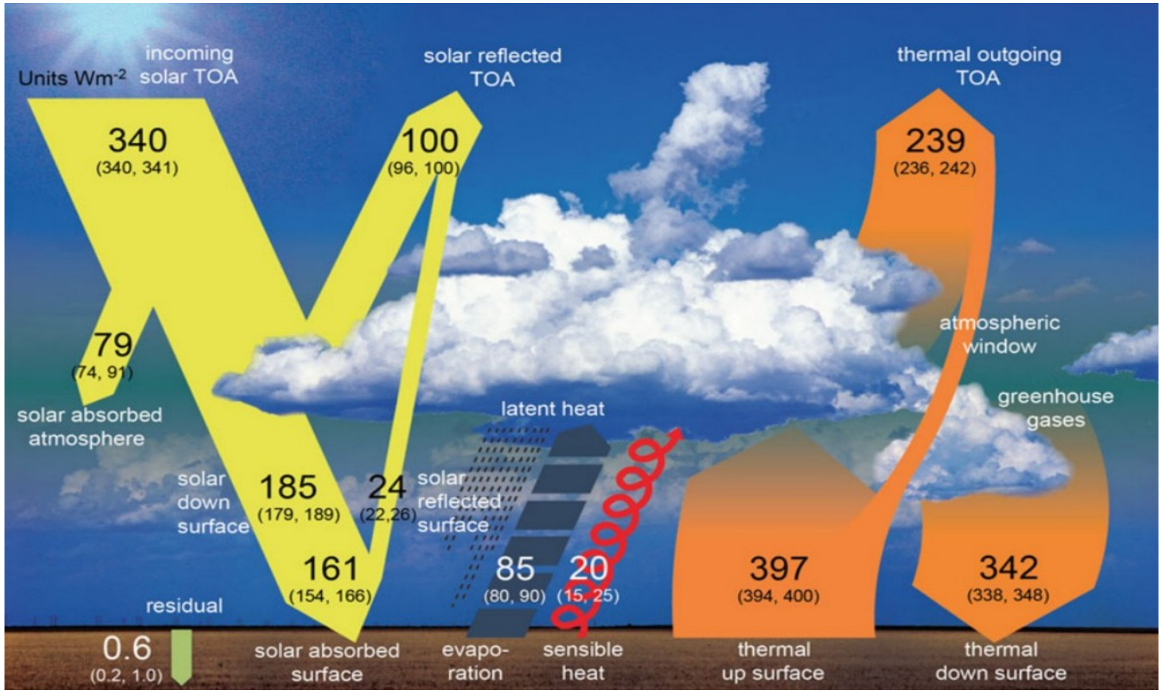
Stratified medium, but :
- Complex physics
- Secular evolution

Climate change: energy balance problem, energy input ≠ energy output.

Current Top of Atmosphere (TOA) energy imbalance (Wild et al. 2012):

estimated at ~0.5 W/m^2 or ~0.15% of 340W/m^2 (TAO input)

→ small fraction of overall radiation → difficult to measure / compute
→ big effect when accumulated over (large) time → climate change



Many ingredients determine energy balance:

- Clouds
- Composition of gas
- Oceans
- Soil coverage / Plants
- Etc., etc.

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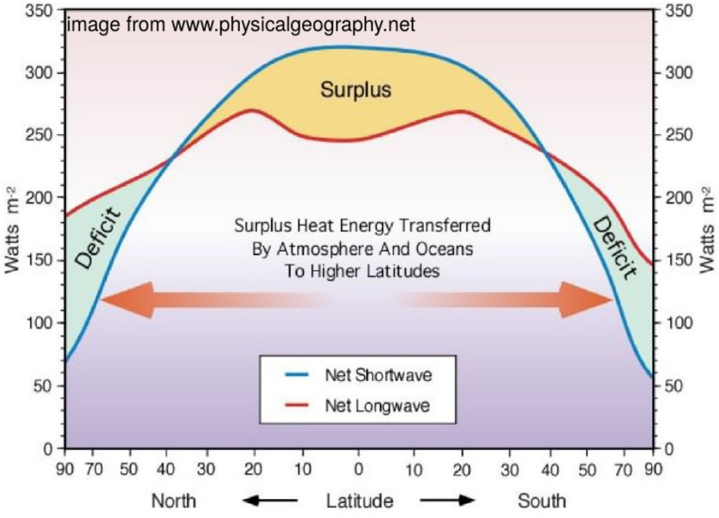
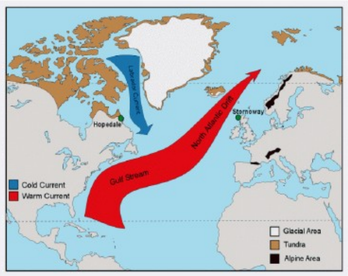
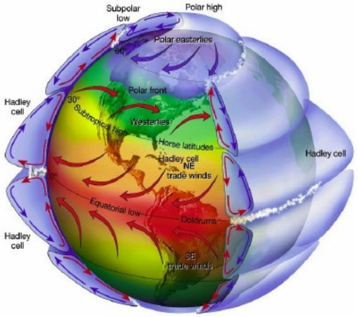
Energy input from Sun is dynamically redistributed

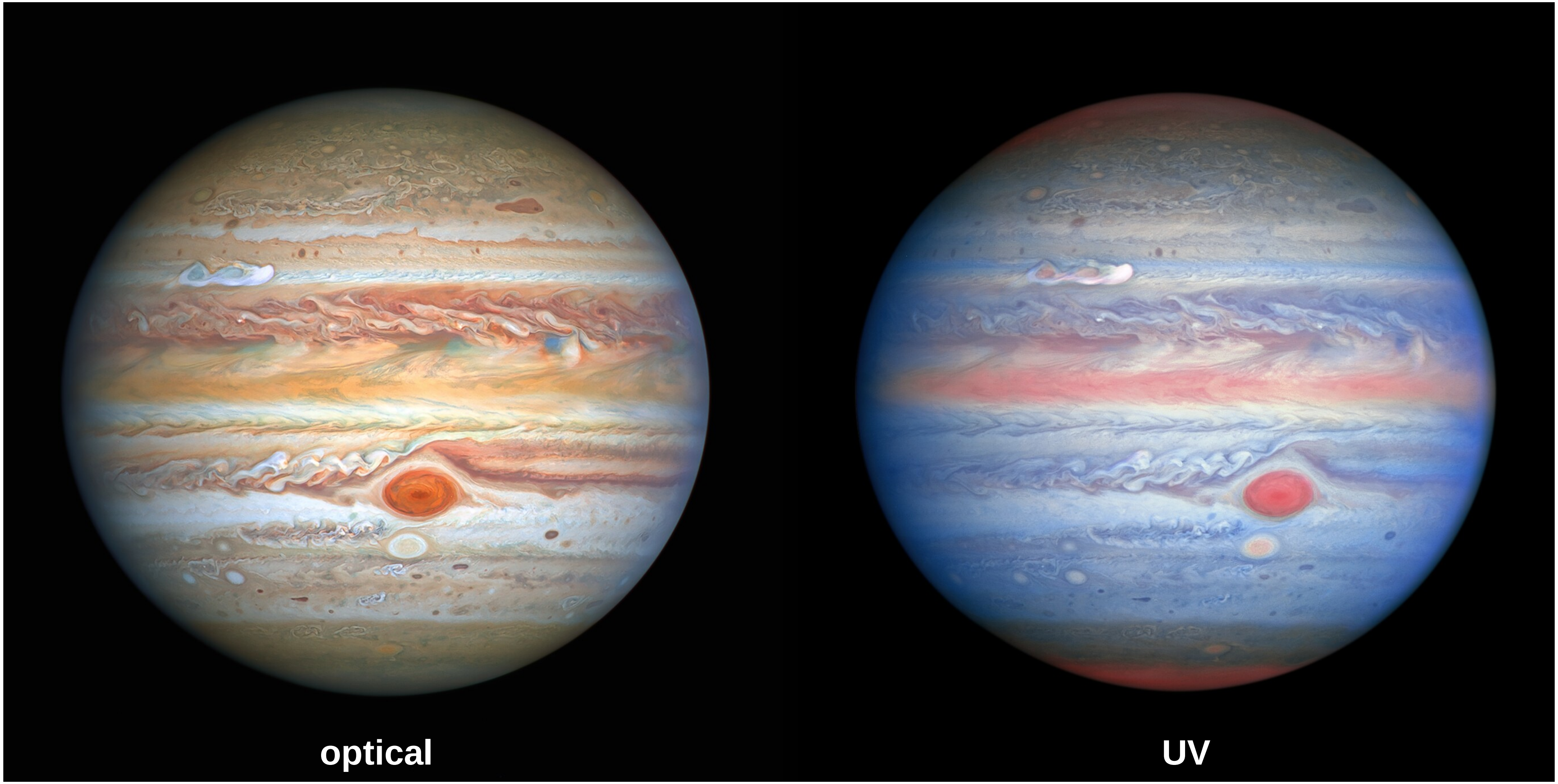
Via atmosphere and ocean currents







Jupiter (representing here planets)



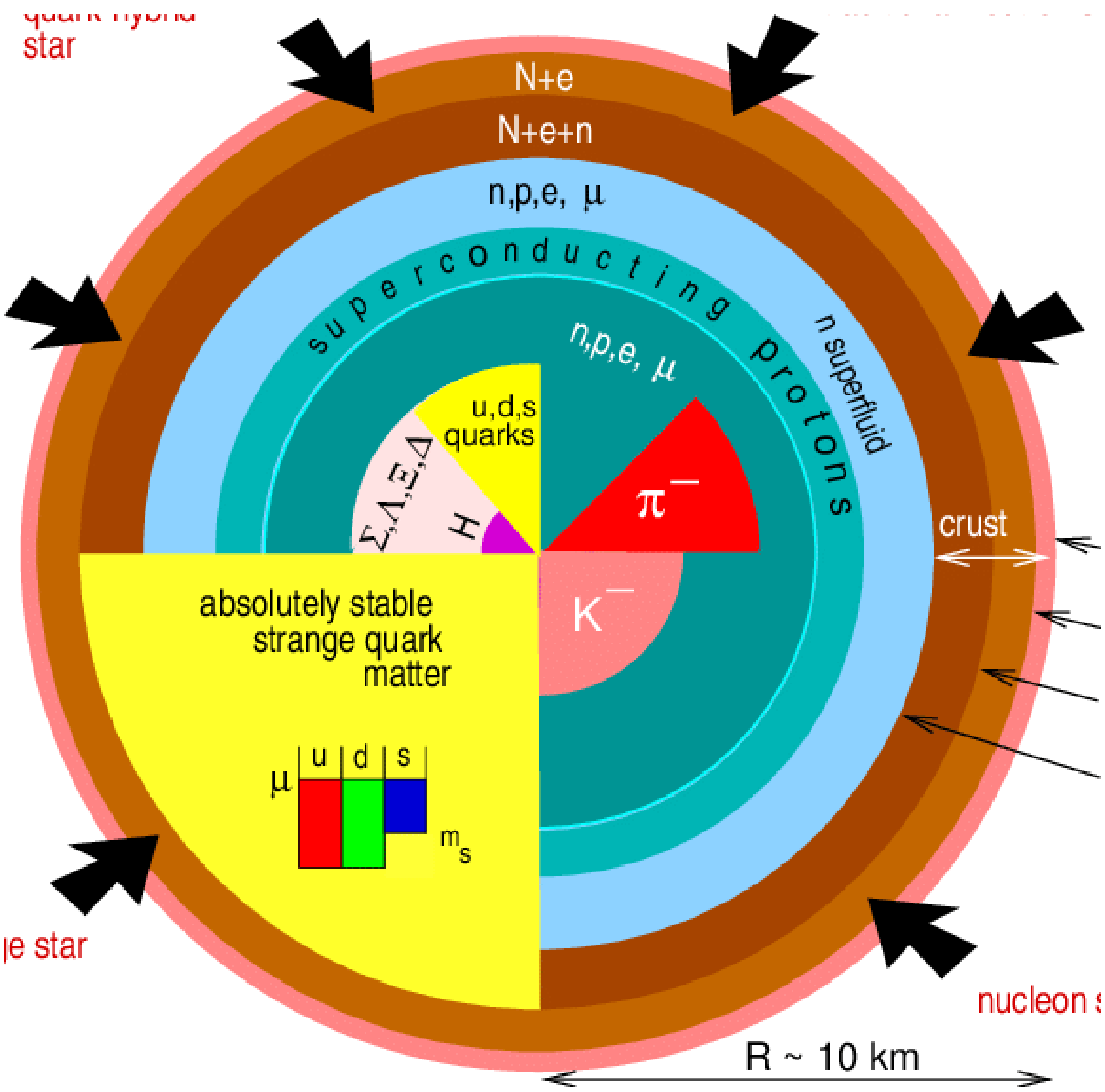
Atmospheric features

- Coherent structures (big red vortex / bands)
- Strong differential rotation with rigid strips

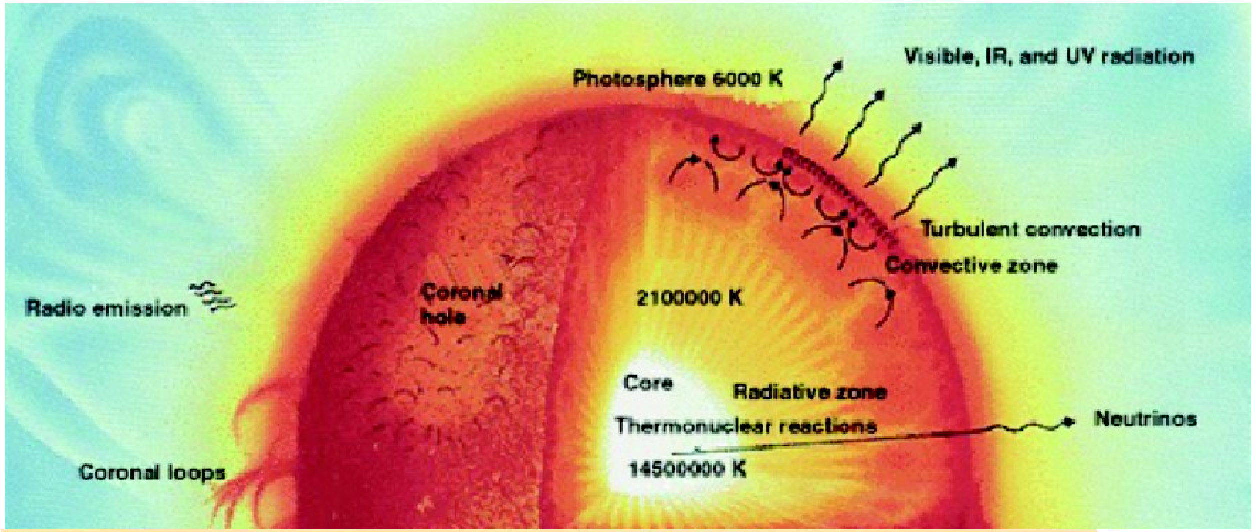
Physical processes

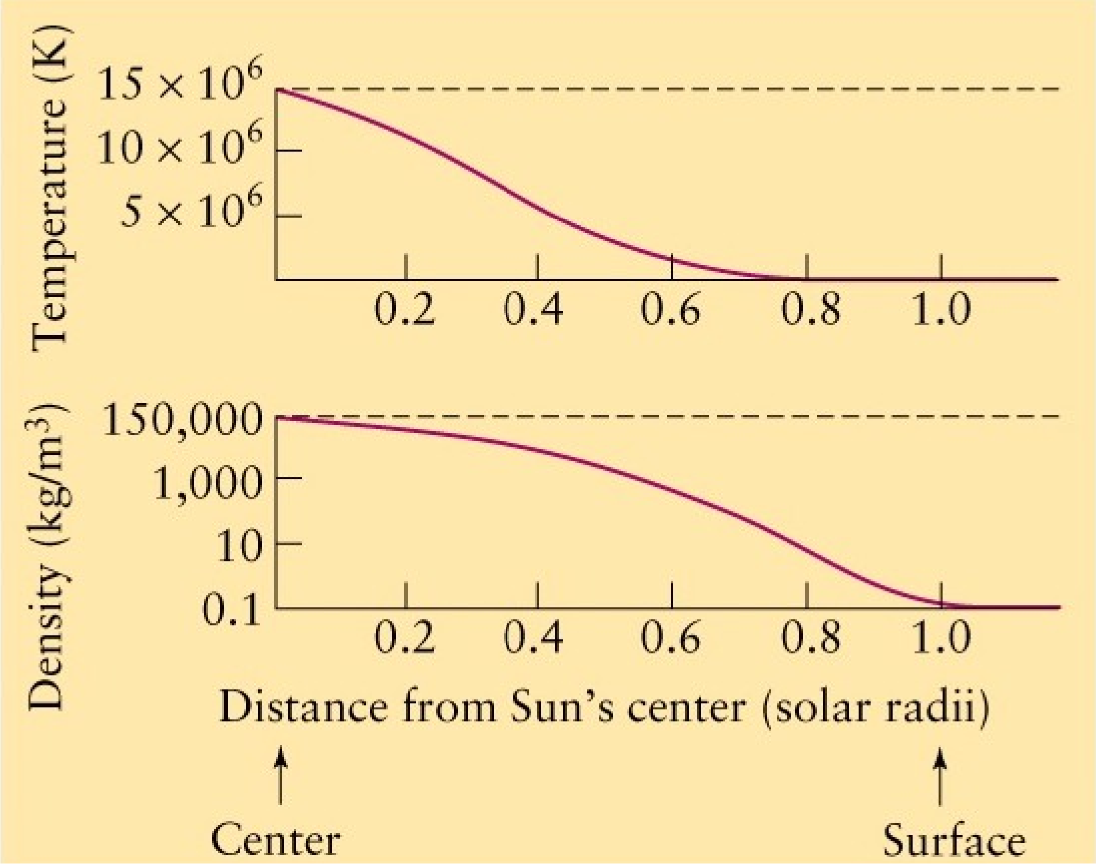
- Strong magnetic/electric fields
- From molecular to ionized gas
- Dust

Compact objects : White dwarfs, neutron stars and black holes



Ordinary stars like the sun : strong stratification





Atmosphere

- Central Temperature
  T_c = 15 million K

T* = 5700 K

- Central Density

  1.5 10^5 kg/m^3

1.5 10^{-6} kg/m^3

- Central Pressure
  $P_c = 10^{11}$ atm

Numerical issues when simulating (large parts) of stars

Characteristics of a stelar flow:

- **Global quasi-stationary equilibrium of stratification**
- **Secular time-scales : stars evolve on a time much larger than any primary time scale:**
  1) **Eddy turn-over time**
  2) **Dynamo action time**
  3) **Nuclear reaction time**
- **Interior: low Mach-number flows: $v/a < 0.05$**
- **Outer atmosphere: supersonic flows/shocks**

We have developed two numerical tools to overcome some of these difficulties

1) A fully implicit, fully compressible code for (M)HD : MUSIC.

2) A well-balanced Scheme for the adaptive mesh numerical tool-kit A-MaZe.

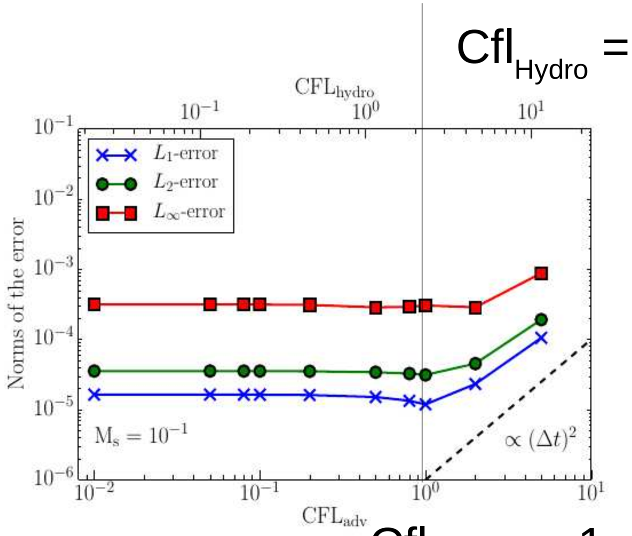
MUlti-dimensional Stellar Implicit Code (MUSIC)

Taylor-Green-vortex : advection errors

10^{-2}

10^{-4}

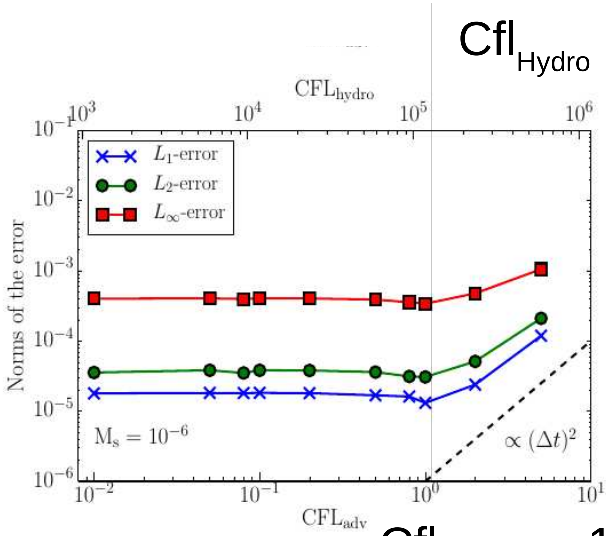
10^{-6}



$\mathrm{Cfl}_{\mathrm{Hydro}} = 20$

$\mathrm{Cfl}_{\mathrm{Advection}}$=1

Machnumber = 10^{-1}



$\mathrm{Cfl}_{\mathrm{Hydro}} = 10^5$

$\mathrm{Cfl}_{\mathrm{Advection}}$=1

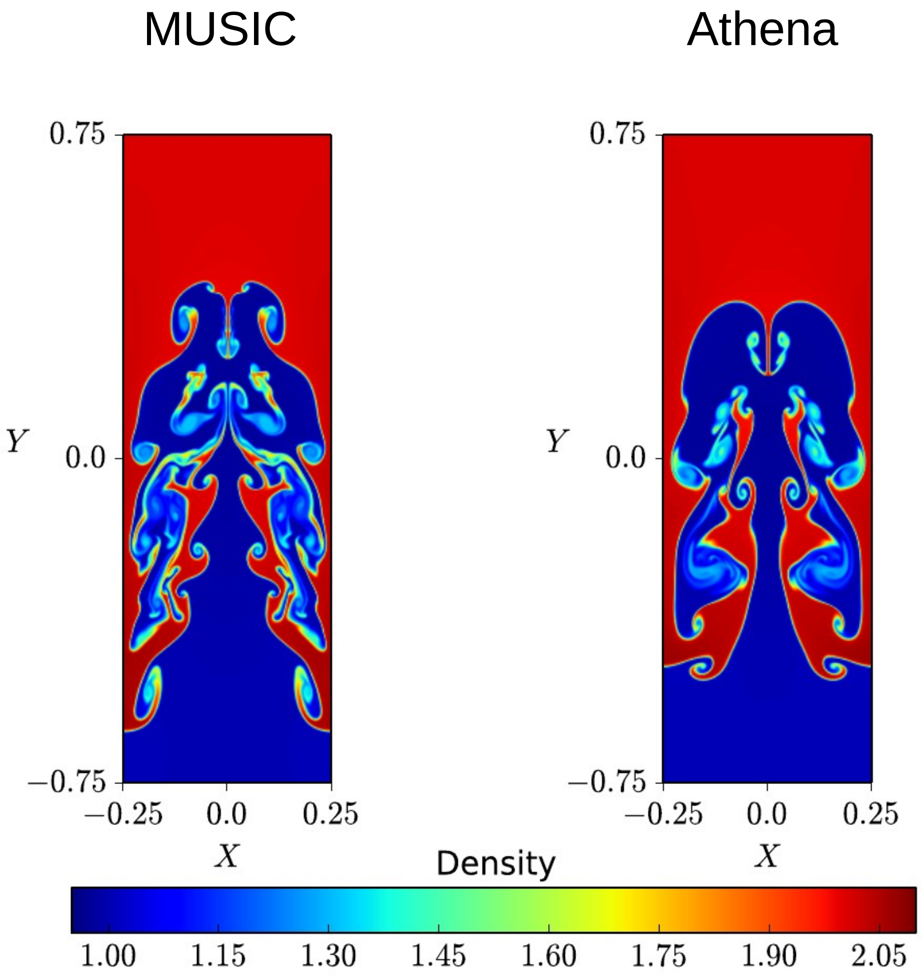
Machnumber = 10^{-6}

Viallet, M.; Baraffe, I.; Walder, R. : *Towards a new generation of multi-dimensional stellar evolution models: development of an implicit hydrodynamic code*, Astronomy&Astrophysics, 531, id.A86, 2011.

Viallet, M.; Baraffe, I.; Walder, R., *Comparison of different nonlinear solvers for 2D time-implicit stellar hydrodynamics*, A&A 555, id.A81, 2013.

Viallet, M.; Goffrey, T.; Pratt, J., Baraffe, I.; Geroux, C.; Popov, M.V.; Folini, D.; Walder, R., *A Jacobian-free Newton-Krylov method for time-implicit multidimensional hydrodynamics,* A&A 586, id.A153, 2016.

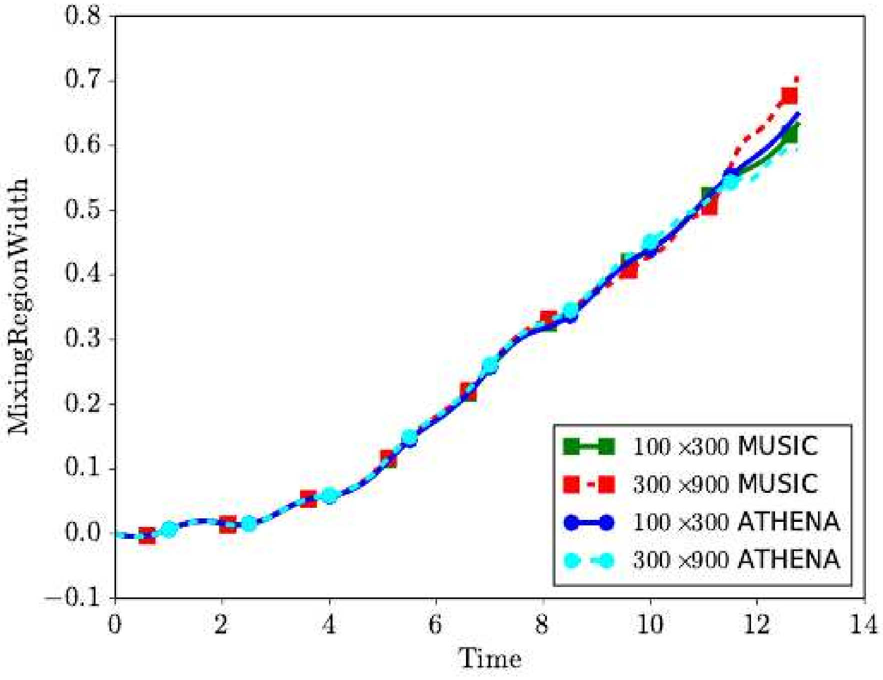
Rayleigh-Taylor-mode : one out of many tests



MUSIC

Athena

- Same height of spike
- Differences in secular instabilities
- Differences in bubble deepness
- Same mixing region/ratio



Goffrey, T.; Pratt, J.; Viallet, M.; Baraffe, I.; Popov, M.V.; Walder, R.; Folini, D.; Geroux, C.; Constantino, T., *Benchmarking the Multi-dimensional Stellar Implicit Code MUSIC*, Astronomy&Astrophysics 600, id.A7, 2017.

A well-balanced scheme for the simulation tool-kit A-MaZe

For a stationary stratification, a necessary condition is
(take momentum Euler equation, $\partial_t = 0$, **v**=0, p : gravitational potential, p: pressure, ρ : density

$$\nabla p = -\rho \nabla \phi$$

This must also be fulfilled numerically, otherwise, a velocity field develops!

Unfortunately, any standard finite volume discretization based on cell-centered variables (ρ, **v**, E) violates the discrete form of this equation !

There many suggestions how to ‘repair’ this. Most of the proposed well-balanced schemes are complicated to implement, work only for a particular EOS, and lead to substantial more CPU costs.

Popov, Walder, Folini, Goffrey, Baraffe, Constantino, Geroux, Pratt, Viallet, & Käppeli
A well-balanced scheme for the simulation tool-kit A-MaZe:
implementation, tests, and first applications to stellar structure
A&A 630, A129 (2019)

have implemented an idea by Käppeli&Mishra (2016) – together with an **essential** modification of the energy equation.

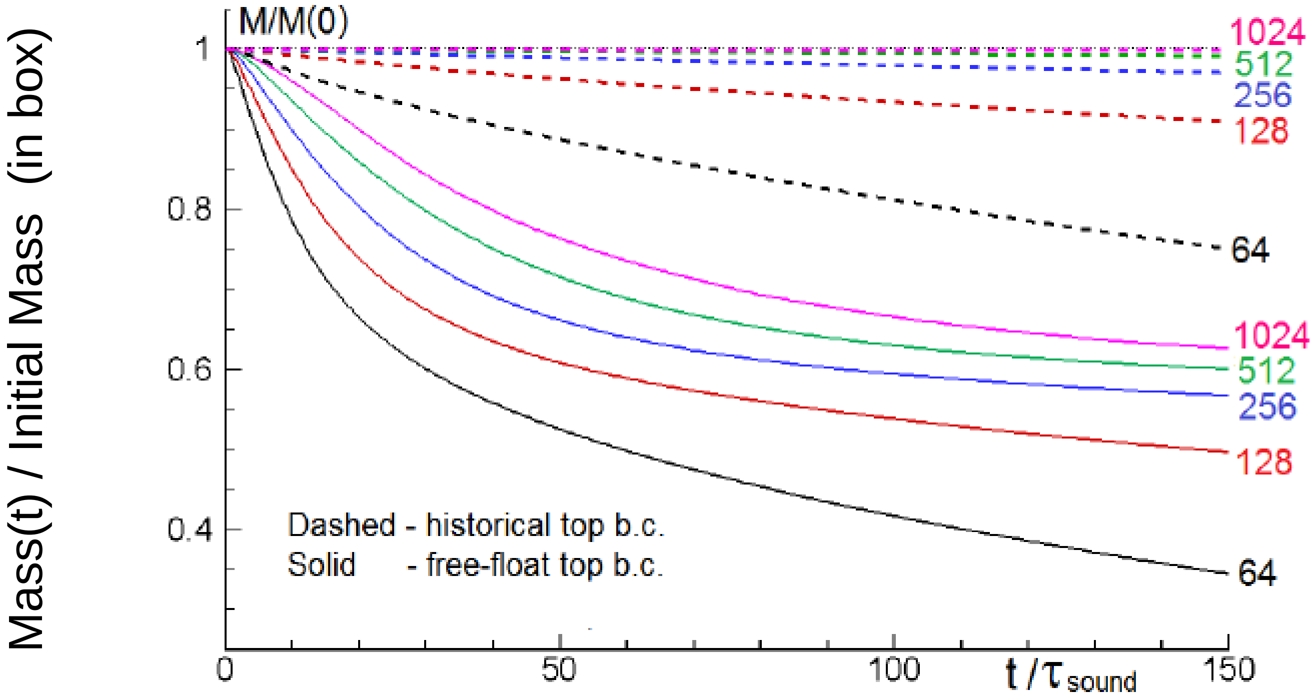
Example 1d : isentropic gas

EOS: $p(\rho, s) = \exp(s/c_v)\, \rho^\gamma$ with $s = s0 = R_{gas} /(\gamma - 1) \ln(p_0 /\rho^\gamma_0)$.

Stationary Solution:
$$\rho(x) = \left(\rho_0^{\gamma-1} - \mathrm{e}^{-s/c_\mathrm{v}} \frac{\gamma - 1}{\gamma} g\, x\right)^{1/(\gamma-1)}$$

WB-scheme : solution is maintained to machine precision

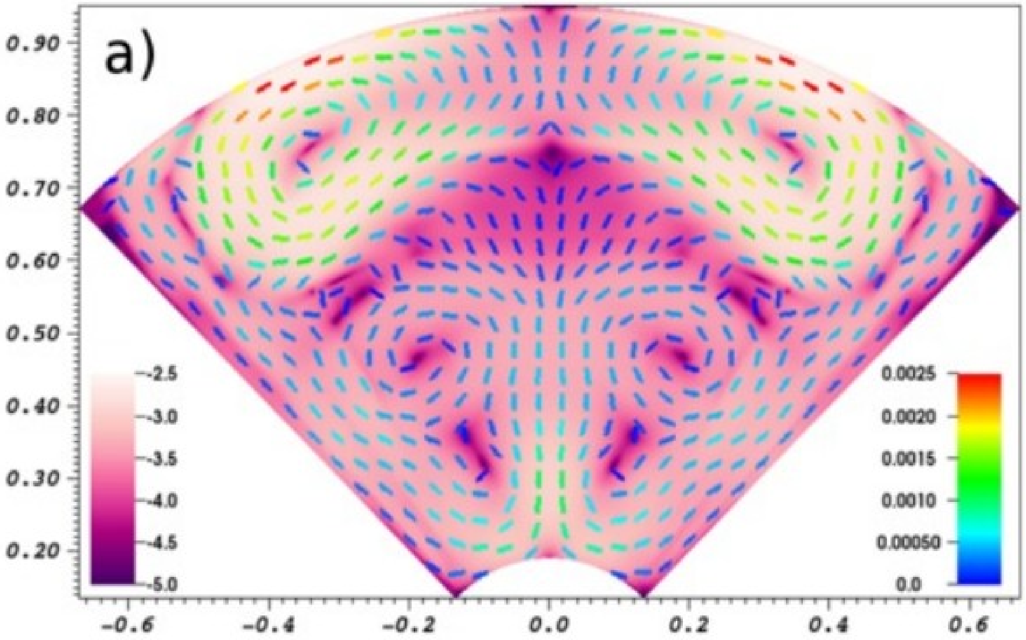
Classical Finite Volume scheme (HHLC Riemann solver, 2nd order in time)



Historical BC
(use exact values in
boundary ghost cells)

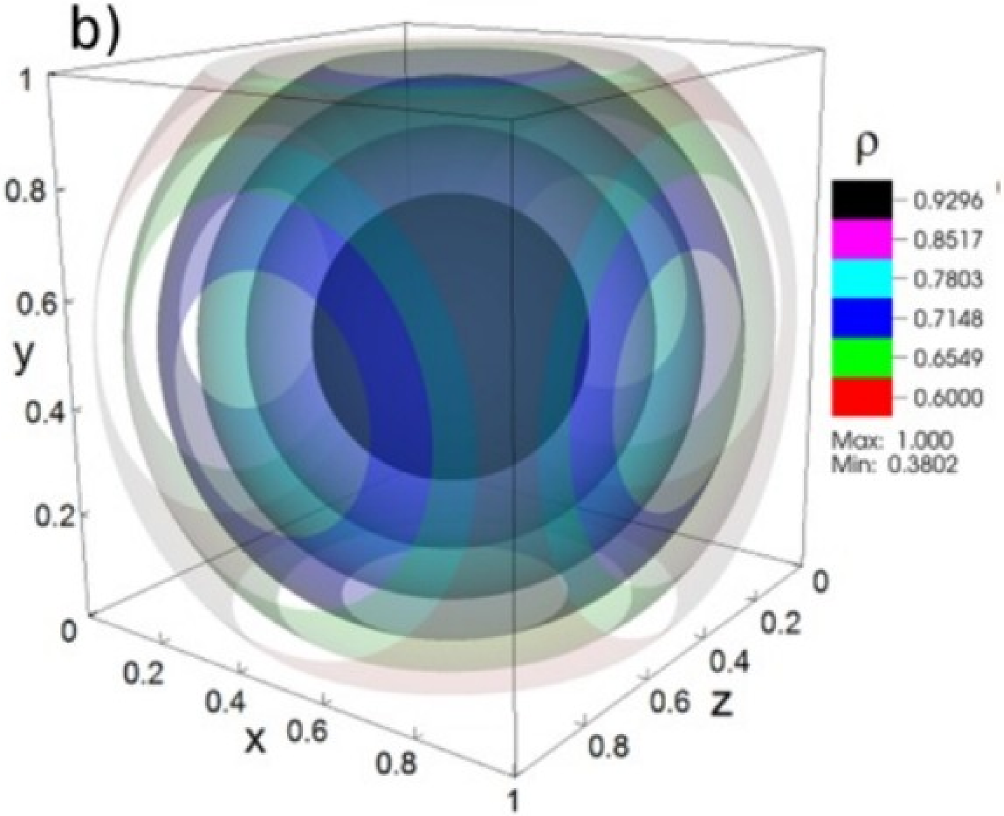
Free flow BC

Lane-Emden Polytrope



Standard scheme:
axi-symmetric 256^2 mesh after 300 τ_s.

A convection-like velocity field develops.
Shown is absolute velocity (from purple to white) with velocity arrows (rainbow colored according to magnitude)
sound speed ranging from 0.76 to 1.39.



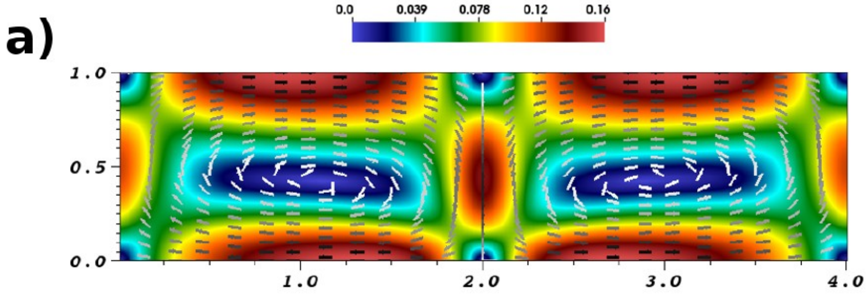
Well-balanced scheme:

3D Cartesian mesh (128^3 , star in a box)

Preserves polytrope to machine precision ($300\ \tau_s$).

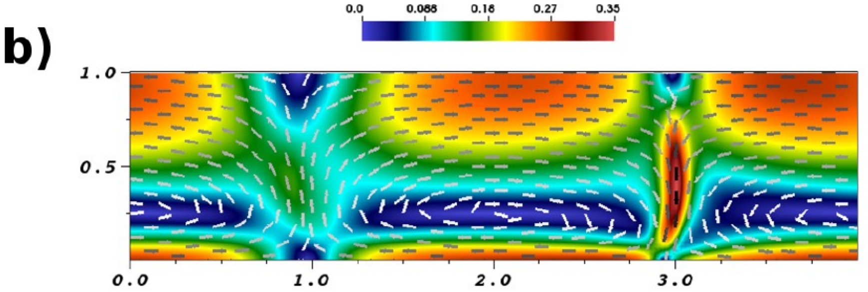
A step towards real atmospheres (test cases of Hurlbert (1984/1986)

- Full, compressible full Navier-Stokes, including heat-transfer by radiation.
- The computational domain is covered by a uniform 2D mesh of $N_x \times N_y = 160 \times 40$.
- Steady convective cells for different stratifications χ ($=\rho_b/\rho_t$)



$X = 1.5$; $R = 310\ R_C$, $R_C = 400$

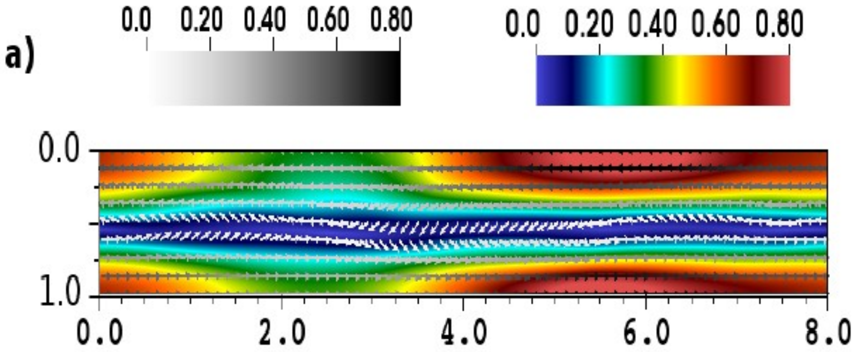
$K_H = 7.1 \times 10^{-3}$, $\mu = 2.8 \times 10^{-3}$



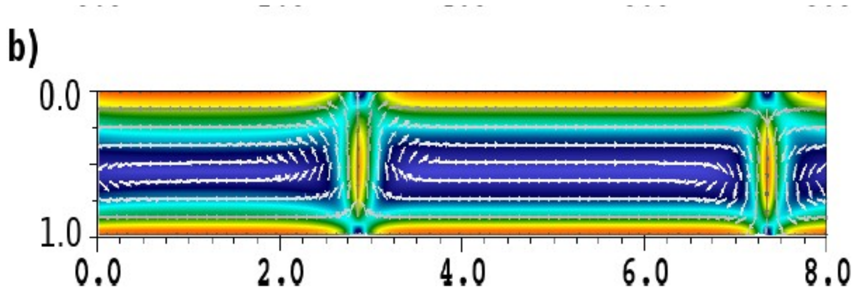
$X = 21$; $R = 1480\ R_C$, $R_C = 750$

$K_H = 1.1 \times 10^{-3}$, $\mu = 4.5 \times 10^{-4}$

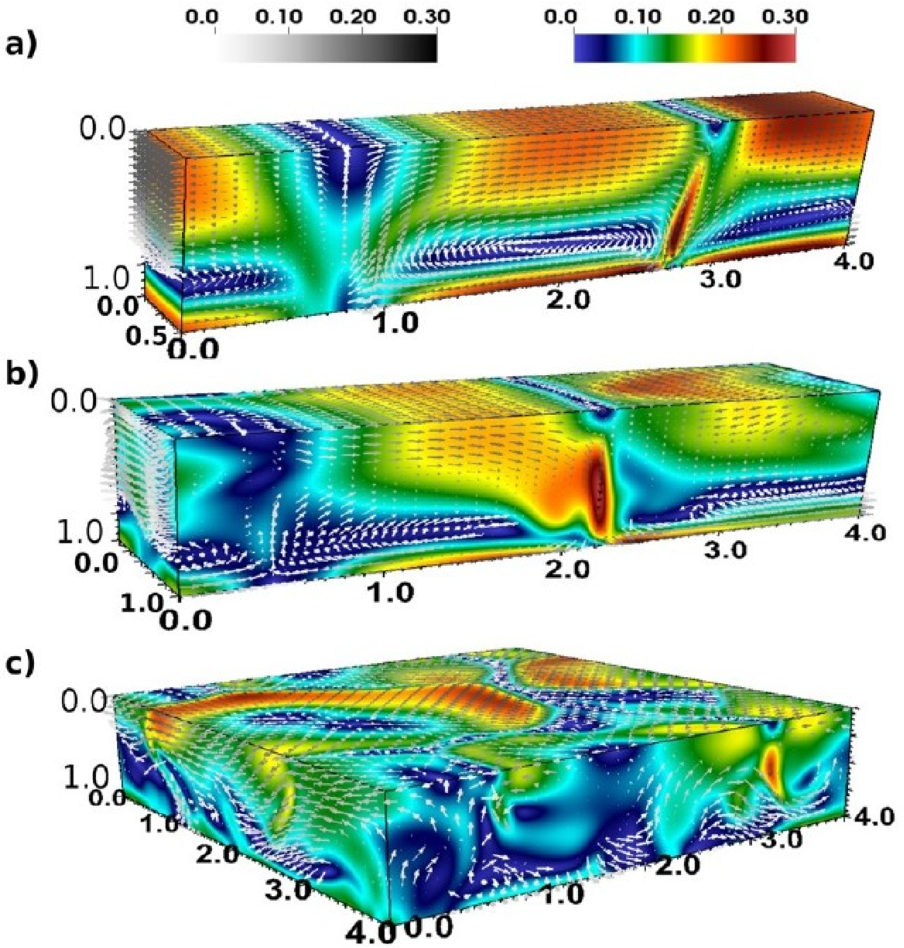
- Zonal flows & convection



$\mu = 1 \times 10^{-10}$

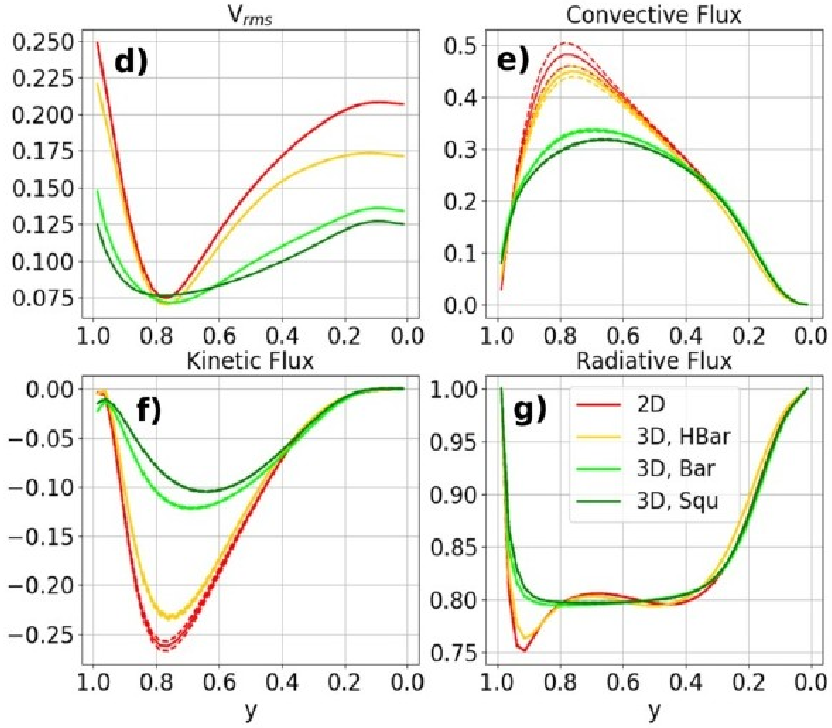


3D Slabs: transition to turbulent convection



X = 21; R = 1480 R_C, R_C = 750
$K_H = 1.1 \times 10^{-3}$, $\mu = 4.5 \times 10^{-4}$

Vertical energy fluxes

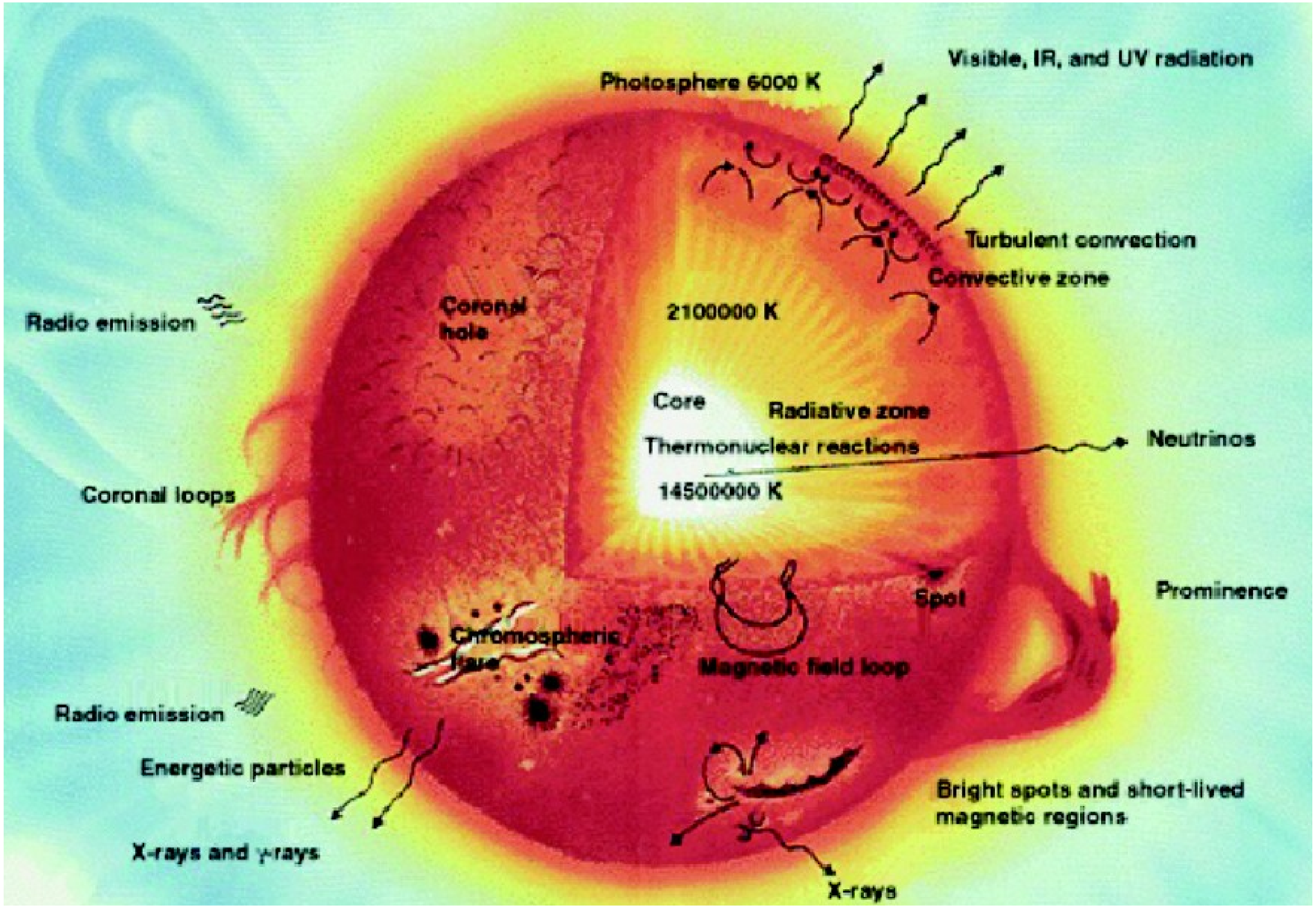


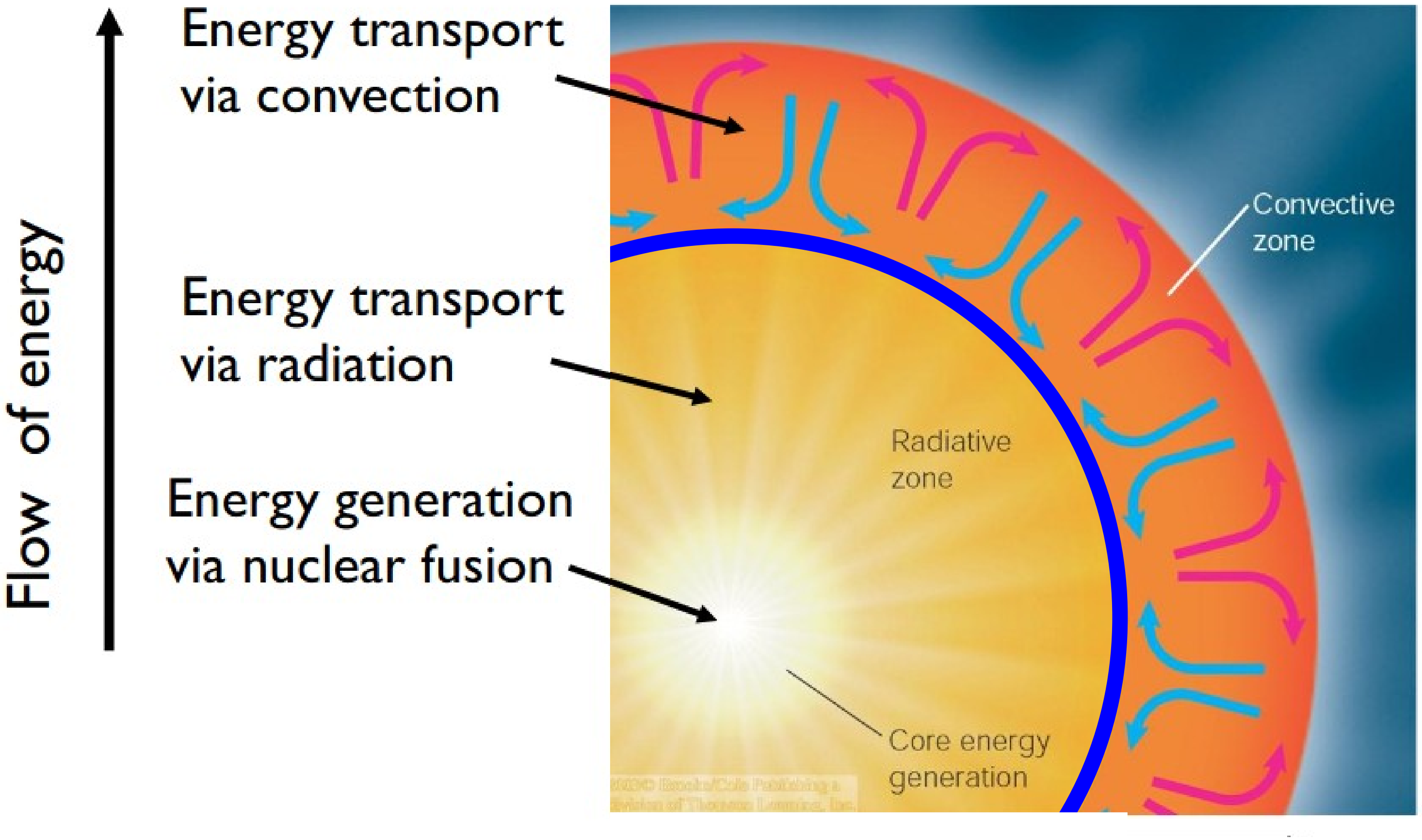
$$F_{\mathrm{C}} = -\overline{c_{\mathrm{p}}\,\rho v_y (T - \overline{T})},$$

$$F_{\mathrm{K}} = -\frac{1}{2}\overline{(\rho v_i v_i) v_y},$$

$$F_{\mathrm{R}} = K\frac{\partial \overline{T}}{\partial y}.$$

Stars in some more details: the Sun





Tachocline

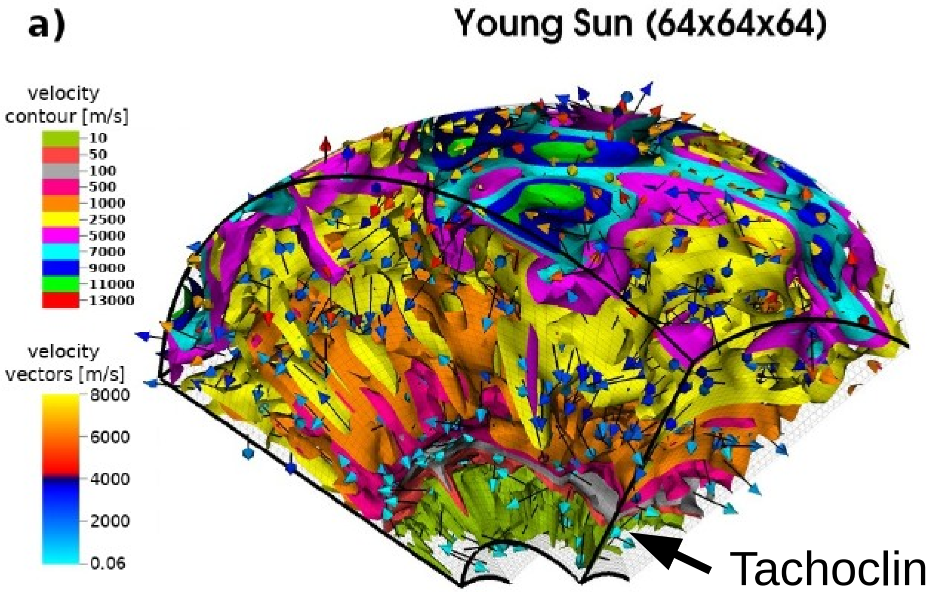
Tachocline : two-fold boundary

- Between the interior region where energy is predominantly transported by photons and the exterior region where energy is transported by convection.
- Between rigidly rotating inner region and lattitude-dependent rotation of the outer region.

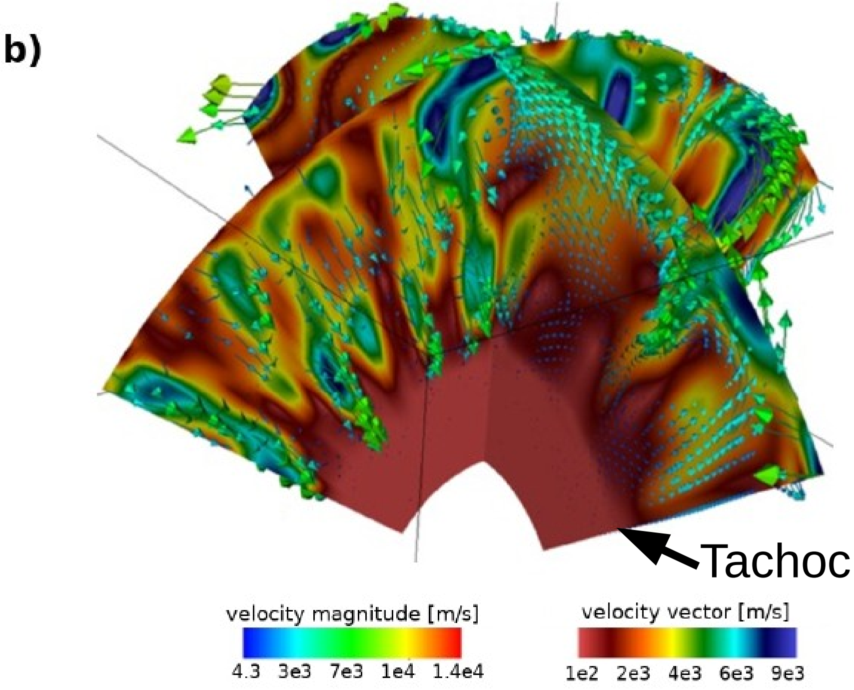
Location/Size :
r/R* = 0.693±0.002 / dr/R* = 0.039±0.013 (Charbannneau et al., ApJ 527, 1999)
dr/R* = 0.019±0.001 (Elliot & Gough, ApJ 516, 1999)

Fully developed turbulent convection

shown: velocity (contours and arrows)

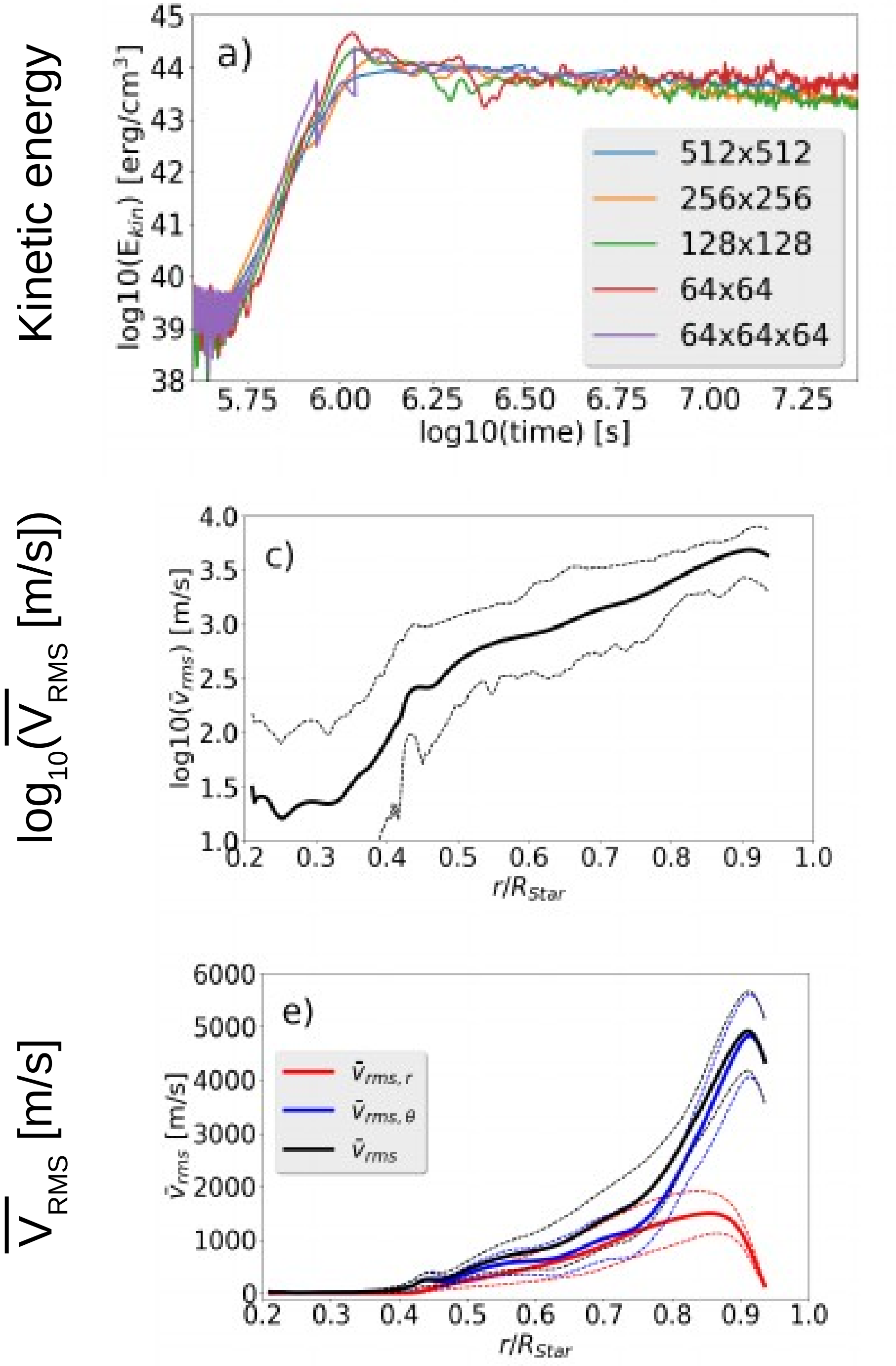


Tachocline



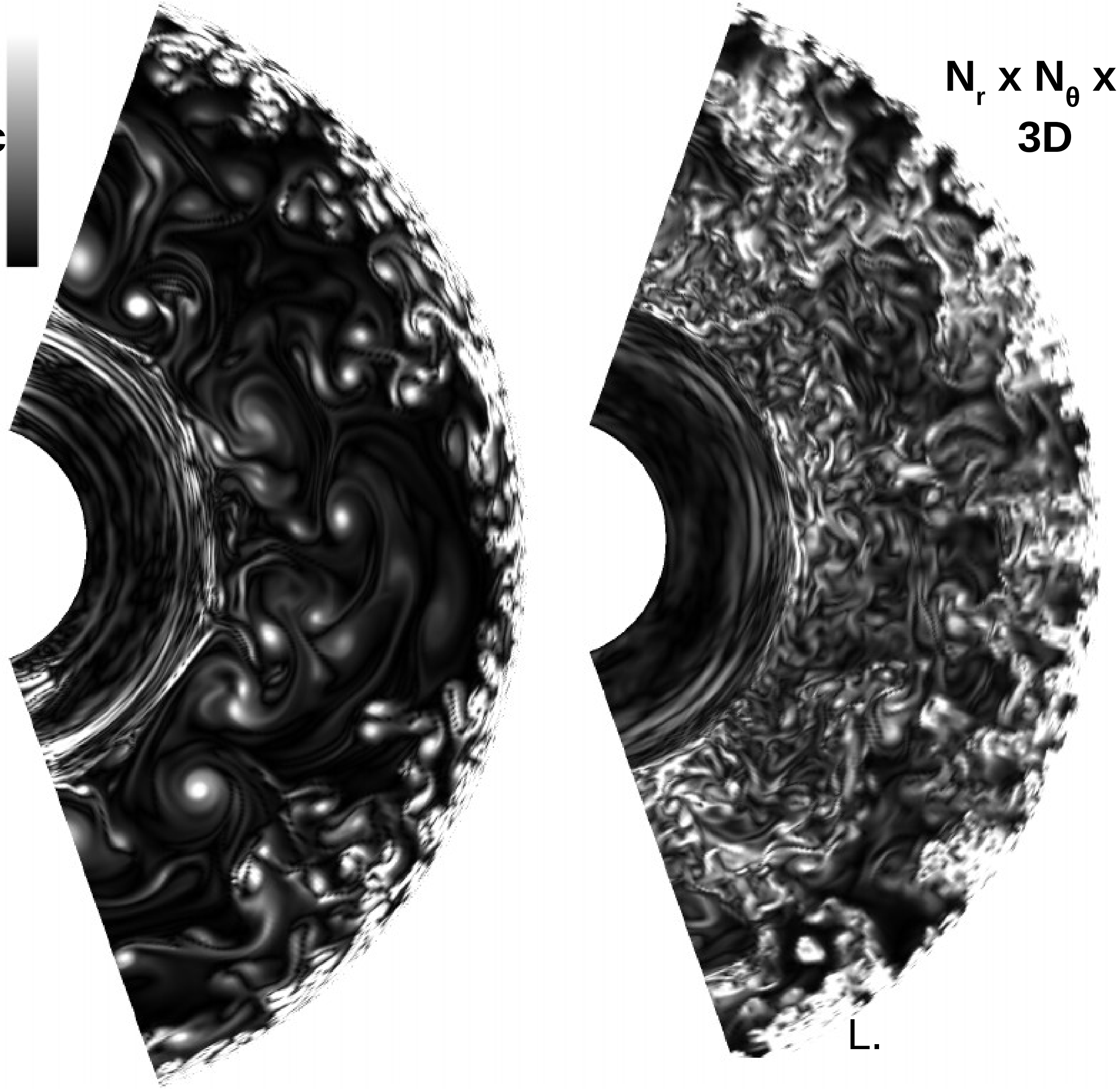
Tachocline

A-MaZe



Study of convective penetration in 2D and 3D (MUSIC)

$N_r \times N_\theta = 780 \times 260$
2D-Axisymmetric



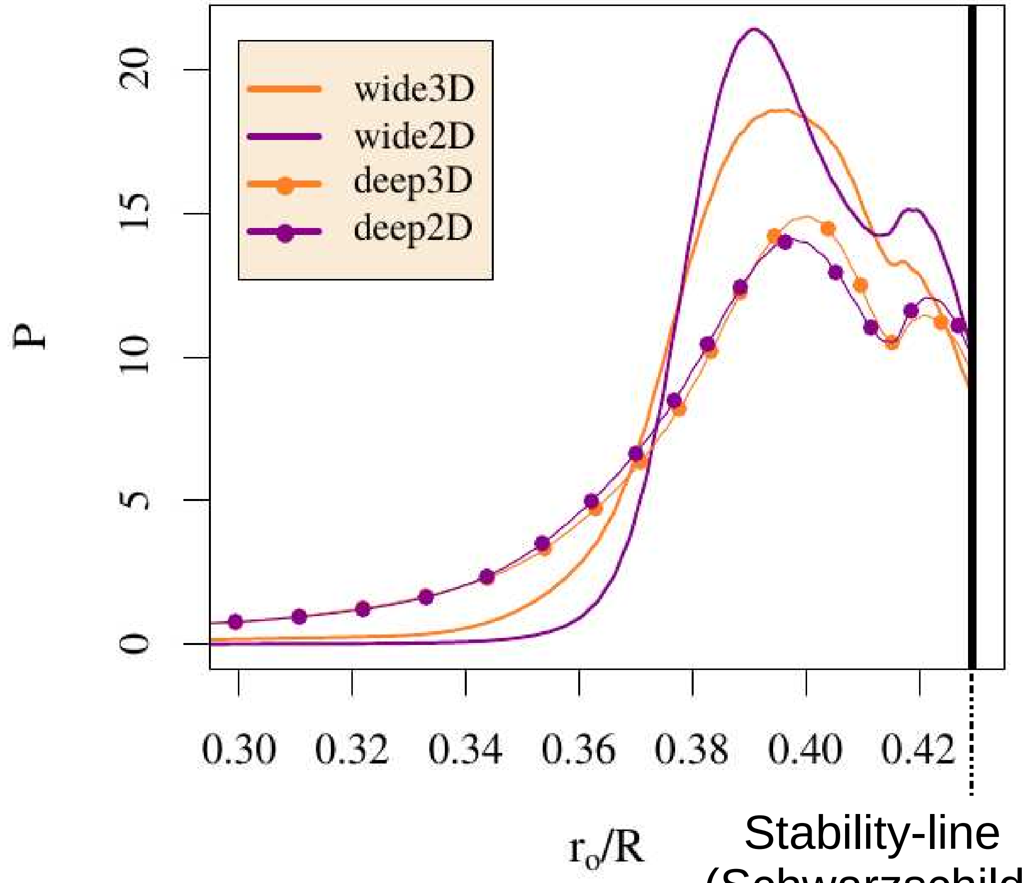
$N_r \times N_\theta \times N_\Phi = 780 \times 260 \times 260$
3D

Vorticity:
lighter color
indicates higher
vorticity magnitude

Pratt et al., *Extreme value statistics for two-dimensional convective penetration in a pre-main sequence star*, **Astronomy&Astrophysics, 604, id.A125, 2017.**

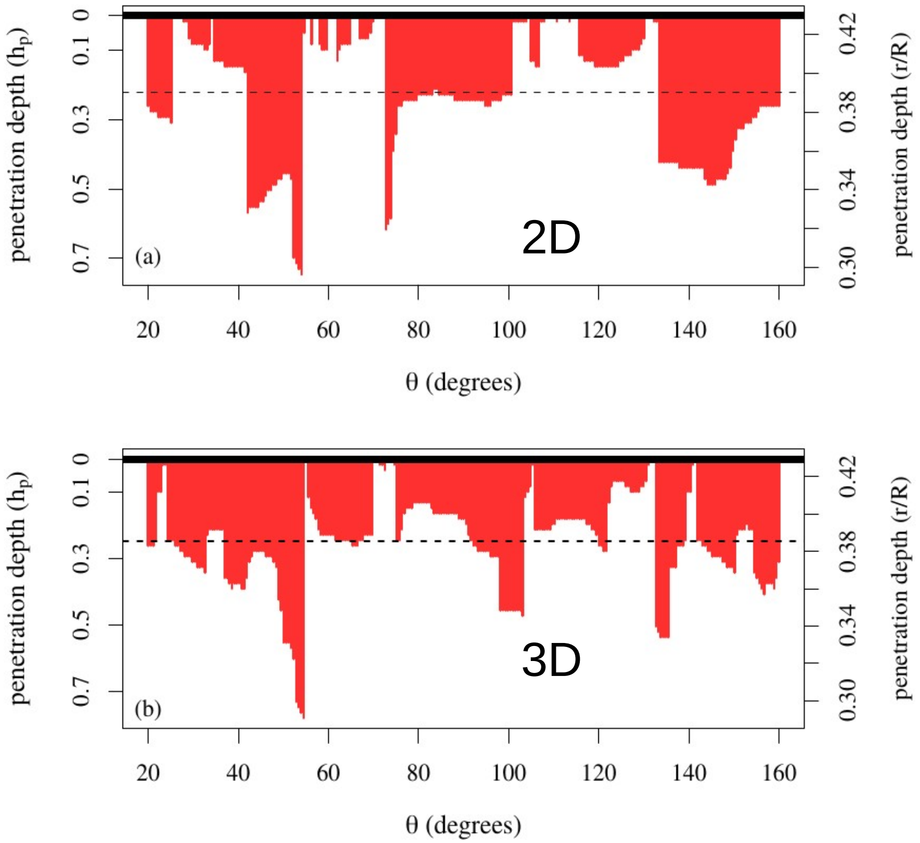
Pratt et al., *Comparison of 2D and 3D compressible convection in a pre-main sequence star,* Astronomy&Astrophysics, 638, id.A15, 2020.

Probability density functions of penetration depth ro determined by the first zero of the vertical kinetic energy flux (10 eddy turnover times)



Stability-line
(Schwarzschild)

Penetration depth at fixed - arbitrary time



Penetration depth : ~ 0.06 R*

Overshooting and Lithium depletion

Long lasting problem:
on the surface of the Sun (as, in fact, of all low-mass stars) one observes that lithium is depleted against of what is observed in solar system meteorites.

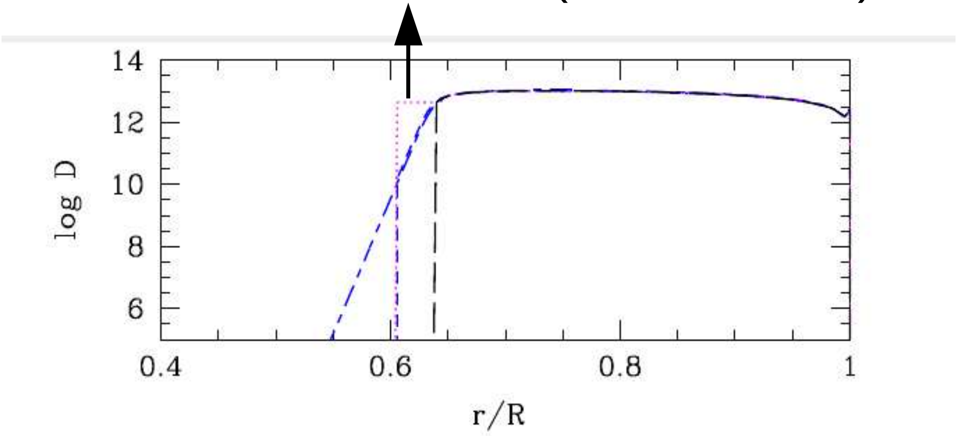
If you look at clusters of low-mass stars, one observes that the depletion depends on the rotation rate of the star: more rotation enforces a stronger depletion.

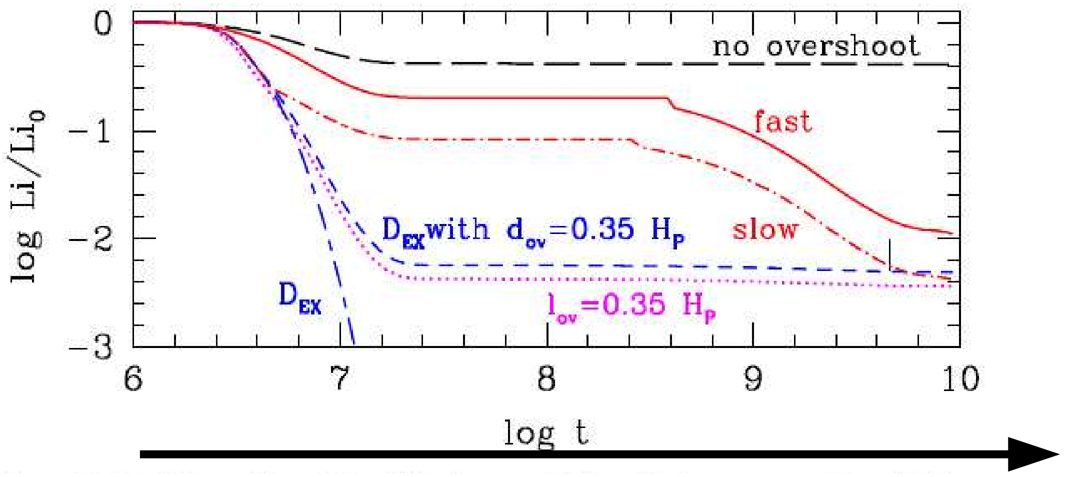
Lithium is extensively burned in the early, pre-main sequence time of stellar life.

Diffusion coefficient D (from 3D Sims) →

1D stellar evolution code

Current
age of Sun





log10(time)

Baraffe et al. *Lithium Depletion in Solar-like Stars: Effect of Overshooting Based on Realistic Multidimensional Simulations*,
Astrophysical Journal 845, id L6, 1017.

Conclusions

- Stratified flows in astrophysics are everywhere present : stars and planets

- Within ERC TOFU, we have developed two performative tools to study such flows:
  1) Fully implicit, fully compressible code MUSIC
  2) Well-balanced module (can be used in any standard cell-centered finite volume code)

- First applications:

  1) Study of model problem with mixed energy transport by radiation / convection

  2) Study of an entire star with an upper convective zone and an interior radiative zone

  3) Overshooting

  4) In combination with 1D stellar evolution code:
     very plausible suggestions of the observed lithium depletion in the solar atmosphere.